WEBB, JARED NEIL, Ph.D. Conceptualizing and Investigating Mathematics Teacher Learning of Practice. (2018)
Directed by Dr. Peter Holt Wilson. 187 pp.

Researchers and teacher educators have made advances in describing mathematics instruction that can support all students in developing conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions toward mathematics. Some scholars have described teaching toward these goals as ambitious teaching – teaching that attends and responds to all students as they engage in intellectually rigorous mathematical activity. To further specify this broad vision, core practices of ambitious teaching are being unpacked and identified so that teachers can learn to enact these practice to support student learning. To support teacher learning, teacher educators have increasingly engaged prospective teachers in rehearsing core practices in less complex settings to learn the skills and purpose for enacting these practices. Emerging research on rehearsals has demonstrated its value in aiding prospective teachers in beginning to enact ambitious teaching practices prior to entering the profession.

While interest in a core practice approach to teaching and teacher learning has grown, scholars have noted that a shared conceptual model of practice might further the field in making progress in accumulating knowledge and building theory of teacher learning of practice. Additionally, others posit that a core practice approach may also support teachers in professional development, yet to this point there has been little conceptual and empirical efforts attending to teacher learning of core practices. This study addresses these gaps in the literature by investigating a conceptual model of

teaching and a teacher educator pedagogy, rehearsal, to advance efforts promoting mathematics teacher learning of ambitious teaching. Three manuscripts collectively illustrate progress on these ideas, drawing upon data and analyses from two years of research in a practice-based professional development for secondary mathematics teachers.

The first manuscript develops and investigates a conceptual model of teaching to improve design and research efforts for teacher learning of ambitious teaching. This conceptual paper addresses a set of design considerations and learning tensions inherent in a core practice approach and examines hierarchical modularity as a way to conceptualize teaching to reconcile these challenges. The second manuscript brings together this conceptual model with a social theory of learning and reports on a retrospective analysis of four teachers' attempts to enact core practices in their classrooms to explore the ways teachers recompose practices over time toward more ambitious forms of teaching. Findings from an analysis of 5,300 instructional moves teachers used over 20 lessons, highlight that small changes in teachers' use of instructional moves that press students to justify their reasoning and orient students to one another's mathematical ideas, supported corresponding changes in teachers' enactments of larger practices of teaching. The third manuscript describes a design for rehearsals for teacher learning of core practices in professional development. It details our design process, describes the ways teachers engaged in rehearsals, and offers evidence of how two teachers engagement in rehearsals corresponded to changes in their classroom practices.

The conceptual arguments in the first manuscript furthers the fields efforts to conceptualize practice to explore teacher learning using a core practice approach. The empirical analysis in the second manuscript provides new ways to explore how learning can be evidenced and investigated across teachers enactments of core practices in their teaching. The design of rehearsals discussed in the third manuscript provides the field with ways to envision and repurpose pedagogies of practice from teacher development to support teacher learning of ambitious teaching. Together, the three manuscripts identify areas for continued inquiry and effort for the design and implementation of practice-based professional development and research on teacher learning of practice.

CONCEPTUALIZING AND INVESTIGATING MATHEMATICS TEACHER

LEARNING OF PRACTICE

by

Jared Neil Webb

A Dissertation Submitted to
the Faculty of The Graduate School at
The University of North Carolina at Greensboro
in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Greensboro
2018

Approved by

_______________________________
Committee Chair

For my Grandpa and Grandma Horse, Fred and Marlene Webb.

APPROVAL PAGE

This dissertation, written by Jared Neil Webb, has been approved by the following committee of the Faculty of The Graduate School at The University of North Carolina at Greensboro.

Committee Chair ___________________________________

Committee Members ___________________________________

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Date of Acceptance by Committee

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Date of Final Oral Examination

## ACKNOWLEDGMENTS

My deepest thanks to:

My advisor, Dr. Holt Wilson, for seeing something in me long before I ever imagined this journey and for investing in my development as a researcher, teacher educator, future university faculty member, friend, and human;

My committee, Dr. Vicki Jacobs, Dr. Paola Sztajn, Dr. Kerri Richardson, and Dr. Jewell Cooper, for their encouragement and confidence that this journey was worth pursuing;

My mentors at UNCG for their thoughtful teaching, advising, and directing throughout my coursework: Dr. Colleen Fairbanks, Dr. Bev Faircloth, Dr. Edna Tan, Dr. Sara Heredia, Dr. Ye He, Dr. Barb Levin, Dr. Dale Schunk, Dr. Heidi Carlone, Dr. Jen Mangrum, Dr. Sam Miller, and Dr. Wayne Journell;

My fellow graduate students: Naomi Jessup, Megan Martin, Arren Duggan, Emily Bryant, Paul Wonsavage, Lauren Baucom, and Faith Freeman, for learning together with me;

My NC State mathematics education friends: Dr. Cyndi Edgington and Dr. Marrielle Myers, for adopting me as a beginning mathematics education scholar;

The teachers and the schools participating in my study, for welcoming me into their lives and sharing their practice: Amy, Marion, Troy, Liz, Rob, Renee, Joy, Tamra, Amber, Joseph, Barbara, Ramona, Daphne, Betty, and Carrie;

My friends, for helping me keep this process in perspective: Matt, Collin, Rich, McCray, Shaun, Mike, Rydell, Leo, and Roger;

My pastors and mentors, Ken and Bettye Bogan, for teaching me how to bring together a love for God and a commitment to racial justice, and for their continued support and understanding as my journey changed;

My parents, Brad & Karolyn, for being amazing parents, loving me unconditionally, and supporting me as I have pursued my dreams;

My brother, Jace, for journeying through life with joy, listening to my nonsense as I processed this journey, and for being a great brother;

My family, for putting up with me and being supportive throughout this pursuit: Ron, Ellen, John, Jill, Michael, Tallulah, Anjani, Kellen, Clayton, and both my Grandma Marlene's;

My kids, Ruby and Bradley, for understanding when "daddy needed to work late" on his "HPD", for being amazing young people, and for challenging me to love others as deeply as I love you both;

My wife, Beth, for being my best friend and partner, for loving and supporting me at my best and my worst, for sacrificing so we can both pursue our dreams, for being an amazing mother to Ruby and Bradley, and for continuing to challenge our family to be better.

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# CHAPTER I

# INTRODUCTION

In this dissertation, I examine a conceptual model of teaching and a teacher educator pedagogy to advance efforts promoting mathematics teacher learning of ambitious teaching. To do so, I draw on hierarchical modularity - a theory to manage complex systems (Simon, 1973, 1996) - as a way to design for and research teacher learning of core practices of ambitious teaching. I propose that such a conceptualization supports recent advances in identifying practices of teaching that support students' learning of mathematics (Core Practices Consortium, 2018; Jacobs & Spangler, 2017), and enhances efforts to design learning opportunities for teachers to rehearse these practices in less complex settings (e.g., Lampert et al., 2013). This dissertation addresses calls for focusing teacher education on the work teachers do in classroom with students around content (Hiebert & Morris, 2012) and research on how teachers improve instruction through participating in professional development and in their classroom teaching (Kazemi & Hubbard, 2008; Sztajn, Borko, & Smith, 2017).

In this introduction, I begin by motivating a focus on teacher learning of core practices of ambitious teaching and defining key terms used in this dissertation. I outline three questions that guided my study and provide an overview of three manuscripts where I detail my investigation and findings. I conclude this introduction by sharing my personal interest in these ideas and the broad significance of this dissertation.

## Motivating a Study Focused on Teacher Learning of Core Practices

Policy documents and education reformers agree that students should learn meaningful mathematics in ways that support the development of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions toward mathematics (National Research Council, 2001). Characterized as “mathematical proficiency”, these goals for student learning can be seen as more intellectually rigorous than goals for student learning focused solely on procedural fluency or individual performance. Intellectually, these goals highlight the need for students to engage in problem solving, leverage and modify their prior conceptions of mathematical ideas, and work toward mathematical understanding that aligns with disciplined forms of school mathematics (National Governors Association Center for Best Practices, & Council of Chief State School Officers [CCSSM], 2010). Socially, researchers and teacher educators have argued that to support all students in learning that leads toward these goals, students should engage in problem solving in ways that foster collective participation and meaningful discourse, and that this discourse should be grounded in students’ mathematical thinking (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Munter, Stein, & Smith, 2015; NCTM, 2014, 2017, 2018; Smith & Stein, 2011).

### Ambitious Mathematics Teaching

Over the past several decades, researchers and teacher educators have made significant advances toward illuminating the kind of mathematics instruction that can meet these goals. Highlighting the forward-looking vision of what mathematics teaching could look like for all students, scholars have organized these efforts around what can be described as *ambitious teaching.* Broadly, ambitious teaching:

> requires that teachers teach in response to what students do as they engage in problem solving performances, all while holding students accountable to learning goals of the discipline - that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions (Kazemi, Franke, Lampert, 2009, p. 1).

For this vision of teaching to meet reform goals for student learning, teachers are encouraged to be proactive and intentional in supporting students by problematizing partial understandings, eliciting and responding to students' individual and collective mathematical thinking, and scaffolding classroom discussions toward formalized learning goals (Kazemi et al., 2009; Munter et al., 2015; Smith & Stein, 2011). Instruction of this kind has been shown to have positive outcomes for student learning toward reform goals (Boaler & Staples, 2008; Franke, Webb, Chan, Ing, & Battey, 2009; Stigler & Hiebert, 2004; Tarr, Reys, Reys, Chavez, Shih, Osterlind, 2008).

**Core Practices of Ambitious Teaching**

Researchers and teacher educators have worked to ensure that this broad vision is further specified by unpacking and identifying *core practices* of ambitious teaching that: occur frequently in teaching; can be enacted using different instructional approaches; allow teachers to learn more about students and the work of teaching; preserve the complexity and integrity of teaching; and are research-based and have impacts on student learning (Grossman, Hammerness, & McDonald, 2009). Recently, a prominent group focused on exploring core practices called the Core Practice Consortium, published their initial definition of core practices, stating that core practices are:

> identifiable components of teaching that teachers enact to support learning. These components include instructional strategies and the subcomponents of routines and moves. Core practices can include both general and content-specific practices (Grossman, 2018, p. 184).

One challenge of a core practice approach to designing for and researching teacher learning apparent in this definition is that core practices can vary in grain size (Jacobs & Spangler, 2017). For example, facilitating a whole-class discussion (Stein, Engle, Smith, & Hughes, 2008) can be seen as a large grain-size core practices, while pressing a student to justify their reasoning (Kazemi & Stipek, 2001) can be seen as a smaller grain-size core practices, or in the case of their definition, a move. In addition, an instructional move has been defined in several different ways, such as “actions meant to facilitate learning typically through a combination of speech and gesture” (Harris, Phillips, & Penuel, 2012, p. 776), and a “unit of teaching activity with respect to a purpose” (Jacobs & Empson, 2016, p. 186), among others.

The design and research efforts presented in this dissertation had a broad goal of honoring teachers’ existing practice and providing opportunities for teachers to rework aspects of their practice toward more ambitious aims for student learning. To do so, our design- research team chose to strategically target three, large grain-size practices that teachers are familiar with, and a few supporting smaller grain-size practices, which I describe as instructional moves to differentiate these practices by grain size. These descriptors provided a way to discuss and work with teachers around these smaller practices by representing them as nested and connected within and across the larger practices and clarifying language in my conceptual model used throughout this dissertation.

Each of the large and small grain-size practices chosen for this study have been sites for research and have been shown to support student learning. In my study, I focus on the larger practices of *launching* a mathematics task (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013), *monitoring* students engagement in a task (Kazemi & Hubbard, 2008), and *discussing* a task with the whole class toward a mathematical goal (Stein et al., 2008). As larger grain-size practices, they entail multiple opportunities for engaging with students, are typically enacted over extended periods of time, and can comprise other smaller grain-size practices. Frameworks for these practices used with teachers in the study are provided in Appendix A. In addition, I focus on two research-based, smaller grain-size core practices, *pressing* students to justify their reasoning or think more deeply about a mathematical idea (Kazemi & Stipek, 2001) and *orienting* students to one another's mathematical thinking (Boaler & Brodie, 2004; Boaler & Staples, 2008; McDonald, Kazemi, & Kavanagh, 2013), which I describe as instructional moves. Definitions and examples of these moves are provided in my codebook in Appendix D.

I chose to focus on pressing and orienting moves because each requires prior interactions with students' mathematical thinking. To use an instructional move, one must attend to both the object of the move (e.g., students' verbal or written externalized thinking) and its goal. For example, to use a probing move, the object of that move might be a student's mathematical idea, and a goal for probing might be to uncover the student's thinking about their approach to a problem. However, using this move does not require prior knowledge of the student's mathematical thinking. In contrast, using a pressing or orienting move requires one to build from an understanding of a student's mathematical thinking. For example, pressing a student to justify their mathematical reasoning requires both a prior understanding of their reasoning and a goal toward which to press, in this case to justify their reasoning. Thus, underlying my study is a broad conjecture that teachers' uses of pressing and orienting moves during enactments of larger practices supports teachers in utilizing students' mathematical thinking to make progress toward their learning goals for students.

**Rehearsing Core Practices of Ambitious Teaching**

A central goal of a core practice approach is to design learning opportunities for teachers to engage purposefully in these practices. To do so, teacher educators have drawn upon Grossman and colleagues (2009) three pedagogies of practice for understanding the practices of professions. *Representations* of the practice of teaching are the ways teachers engage with images of teaching made visible by teacher educators, such as through model lessons or video cases of teaching. *Decompositions* of the practice of teaching are the ways teachers engage in deconstructing practice to highlight and specify particular aspects of teaching, such as launching a mathematics task, noticing students' mathematics, or attending to issues of equity. *Approximations* of the practice of teaching refer to learning opportunities for teachers to engage with and practice important aspects of teaching through activities such as analyzing student work or rehearsing.

Teacher educators interested in supporting prospective teachers using a core practice approach have increasingly draw on an approximation of practice called rehearsal. Used within ongoing cycles of investigating and enacting core practices, *rehearsal* is a teacher educator pedagogy in which teachers prepare and rehearse contingent and interactive core practices of teaching with their peers or students while receiving feedback from teacher educators (Lampert et al., 2013). One overarching goal of engaging prospective teachers in rehearsal is to address the well-documented disconnect between prospective teacher education, the complexities of teaching in schools, and critiques that teacher preparation has been too focused on the knowledge needed to teach, with less attention on how to use this knowledge in practice (Ball & Forzani, 2009; Forzani, 2014; Grossman et al., 2009). Emerging research on the use of rehearsal and a core practices approach has demonstrated its effectiveness in supporting prospective teachers in understanding and learning to enact ambitious teaching practices as they begin their teaching careers (e.g. Boerst et al., 2011; Campbell & Elliot, 2015; Han & Paine, 2010; Hunter & Anthony, 2012; Kazemi, Ghousseini, Cunard, & Turrou, 2016; Lampert et al., 2013).

**Supporting Teacher Learning in Professional Development with Rehearsals**

Teacher educators and researchers interested in supporting teacher learning of practice in professional development have also drawn upon the pedagogies of representing, decomposing, and approximating the practices of teaching (Ball & Cohen, 1999; Silver et al., 2007; Wilson & Bern, 1999). Structured around artifacts of teaching such as student's written work (Kazemi & Franke, 2004), classroom video (van Es &

Sherrin, 2006), or clinical interviews with students (Jacobs & Empson, 2016; Wilson, Mojica, & Confrey, 2013), teacher educators have used these artifacts as part of professional learning tasks to make practices of teaching public for approximation (Silver, Clark, Ghousseini, Charolambous, & Sealy, 2007; Smith, 2001). These approaches have been shown to support teacher learning; however, some teacher educators encourage approximations that are more proximal to the in-the-moment work of teaching, such as rehearsals, to provide opportunities to deliberately try out core practices in ways that more closely align with the in-the-moment complexities of teaching (Grossman & McDonald, 2008; McDonald et al., 2013; Sandoval, Kawasaki, Cournoyer & Rodriguez, 2016).

While encouraged for many years, to this point, there has been little theoretical, conceptual, and empirical work attending to teacher learning of core practices, their engagement in rehearsals of these practices in professional development, or their attempts to enact them in their classroom teaching (Kazemi & Hubbard, 2008). In addition, while we have made progress in both teacher preparation and teacher professional development, we need conceptual tools to attend to the complexities of practice; and a shared conceptualization of practice would support the field in making progress in accumulating knowledge, building theory, and developing shared language of practice that might support teacher learning (Clarke & Hollingsworth, 2002; Opfer & Pedder, 2011). This dissertation serves as a concerted effort to contribute to both foundational and emerging research on a core practices approach to supporting teacher learning and builds upon research on teacher learning in professional development. Moreover, it serves to introduce a conceptual model for practice that can both address challenges of a core practices approach and further efforts to build theory and accumulate knowledge of teacher learning of practice. To do so, I ask the following questions:

1) How can teaching be conceptualized to inform research and design for teacher learning that both respects and challenges teachers' existing practices?,
2) In what ways do teachers recompose cores practices together across their participation in two years of professional development focused on practices of ambitious teaching?, and
3) What is a design for rehearsals in professional development that supports teachers in learning core practices of ambitious teaching?

To answer these questions, I present findings from a retrospective analysis of a multiyear professional development design study organized as three manuscripts. Across these manuscripts I detail a conceptual model for practice and use this model to investigate changes in teachers' classroom practices, teachers' engagement in rehearsals of practices in professional development, and ways in which these changes might relate to teachers' engagement in rehearsals. The next section provides an overview of the study and a summary of each manuscript, highlighting distinctions that warrant three separate pieces that together represent the entirety of this study.

## Overview of the Study as Three Manuscripts

The study took place over two implementations of a practice-based professional development with secondary mathematics teachers. I was involved as a facilitator, design-team member, and researcher for both implementations. The impetus for this study began in 2014, when our research team began to consider the ways we could incorporate rehearsals into professional development for teachers and explore the relationship between teachers' participation in rehearsals and their enactments of these practices in their classroom teaching. Data for this study were collected from the 2015 and 2016 summer institutes of our professional development and from teachers' classroom lessons across five academic semesters from 2015-2017. These data include video recordings and transcripts of five classroom lessons and rehearsals in the summer institutes for participating teachers, as well as data from focus groups, debriefings, and teachers' written reflections. The three manuscripts that comprise this dissertation draw upon different subsets of these data.

The first manuscript, titled "Conceptualizing Practice for Teacher Learning in Professional Development", is a conceptual paper that addresses the question: *1) How can teaching be conceptualized to inform research and design for teacher learning that both respects and challenges teachers' existing practices?* The paper introduces and examines a conceptual model of teaching using a core practice approach. It addresses a set of design considerations (Jacobs & Spangler, 2017) and learning tensions (Jansen, Grossman, & Westbroek, 2015) of a core practice approach and examines hierarchical modularity (Simon, 1996) as a way to conceptualize teaching to reconcile these challenges. I use teachers' engagement in rehearsals and their classroom teaching as context to provide examples that support my theoretical analysis. I propose that hierarchical modularity is a way for researchers and teacher educators to both design for and research teacher learning of practice in ways that can further accumulate knowledge and build theory of teacher learning of practice.

In part, this manuscript highlights the need for attending to the ways teachers bring multiple practices together over time to teach in ways that are more responsive to, and supportive of students' individual and collective learning. While mathematics teachers and researchers have taken up decomposing teaching into core practices to support teacher learning, Jansen and colleagues (2015) argue that little attention has been given to the complement of decomposing practice – *recomposing* practice back together to support more ambitious teaching. While investigating and learning individual practices is important, a lack of attention to the ways teachers recompose multiple practices together may hinder both teachers' learning and enactments practices as well as researchers understanding of teacher learning of multiple practices. To address this tension, I take on my second question of, *In what ways do teachers recompose cores practices together across their participation in two years of professional development focused on practices of ambitious teaching?*, in two ways.

In the first manuscript, I provide examples of how teachers recomposed three large grain-size core practices together to enact whole lessons. In a second, empirical paper titled, "Secondary Mathematics Teachers' Recompositions of Core Practices of Ambitious Teaching", I take up this consideration from a different perspective, retrospectively analyzing four teachers attempts to enact the large grain-size practices of launching, monitoring, and discussing. Findings from the analysis highlight the ways small changes in teachers' uses of pressing and orienting moves, which represented 11%

of the 5,300 moves coded across teachers' lessons, supported corresponding changes in teachers' practices of launching, monitoring, and discussing. These findings emphasize how small changes in the use of and goals for instructional moves can have profound impacts on instruction.

Across the first and second manuscripts, I briefly describe and draw upon data from teachers' engagement in rehearsals in the summer institutes of our professional development. In the final paper written for practitioners titled, "Designing Rehearsals for Secondary Mathematics Teachers" – I address my third question of, *What is a design for rehearsals in professional development that supports teachers in learning core practices of ambitious teaching?* To do so, I use findings from the second manuscript to motivate the need for exploring the relationship between changes in teachers' enactments of core practices in their classrooms and their engagement in rehearsals. I describe rehearsals for professional development with secondary mathematics teachers by detailing our design process, presenting the ways teachers engaged in rehearsals in professional development, and providing evidence of how engaging in rehearsals supported two teachers in imagining new ways of teaching that aligned with changes in their classroom practice. I conclude this manuscript with design considerations, revisions to our rehearsals, and discuss the role of mathematics teacher educators in supporting teachers working to improve their practice.

## Highlighting the Intersecting Role of Researcher and Teacher Educator

As a mathematics teacher educator and researcher, I am continually designing for learning and learning from design. I came to this study with a broad and rich set of experiences across my graduate career, but also as a relative novice at both designing for and researching mathematics teacher learning. My early experience as a graduate research assistant for the *Learning Trajectories Based Instruction* project, led by Drs. Paola Sztajn and Holt Wilson, served as a foundation for being prepared for this dissertation study.

First, I was able to experience the complexities of designing for teacher learning in professional development and the rigor necessary for design-based research. From this, I was better equipped to attend to the design of rehearsals used in this study, the modifications made across implementations, and researching the ways teacher learning could be investigated both in the professional development and in teachers' classrooms. Second, I was able to experience the process of retrospectively tailoring broad, initial research questions and analyzing teacher learning longitudinally across their participation in professional development and teaching. From this, I was better prepared to develop the overarching focus of this retrospective study and determine what I was able to explore and what was outside the scope of data available to conduct my analyses. Third, as a part of one retrospective study, I had the opportunity to experience the rigorous theoretical work necessary to bring together multiple frameworks to answer research questions (Wilson, Sztajn, Edgington, Webb, & Myers, 2017). This experience was foundational to bring together hierarchical modularity (Simon, 1973), a theory typically used in research on physical systems, with a social perspective of learning (Wenger, 1998) to present a conceptual model for practice that maintained a focus on teaching and teacher learning as a social endeavor.

A second experience central to this dissertation was the pilot design and research efforts led by my dissertation advisor and mentor, Dr. Holt Wilson. In the fall of 2014, he introduced me to the emerging research on rehearsals used in preparing prospective elementary mathematics teachers (Lampert et al., 2013). This interest led to two pilot studies that served as the foundation for my dissertation research. First, in the spring of 2015, as a part of a design-based research doctoral seminar course, I worked with other doctoral students to develop and pilot rehearsals for practicing secondary mathematics teachers around the practice of monitoring. Building from that experience, in the summer of 2015, Dr. Wilson and I designed a practice-based professional development in which we incorporated rehearsals for the practices of launching and monitoring. Across these two cycles of pilot work, I continued to refine the skills needed for this dissertation.

## Significance of Dissertation

As my interest in designing and researching teachers' engagement in rehearsals and their enactment of core practices in their teaching progressed across these experiences, I also deepened my appreciation of the complexity of teaching mathematics and my commitment to honoring teachers. Mathematics teaching is complex, and the work that teachers do to manage and reconcile multiple commitments and goals for their own teaching, in their own context, is always inspiring. From the outset, the contributions I hoped to make within this dissertation built from this commitment.

The conceptual model put forth and investigated in the first manuscript makes meaningful progress in efforts to support and understand teacher learning using a core practice approach. The empirical analysis in the second manuscript builds from the first manuscript and offers empirical evidence of teacher learning of core practices using this conceptualization of practice. The rehearsals discussed and explored in the third manuscript outline new ways to envision and repurpose pedagogies of practice typically used with prospective teachers for teachers in professional development. The sum of the three manuscripts brings into focus areas for continued inquiry and effort for the design of practice-focused pedagogies, research of the impacts these pedagogies can have on practice, and ways to respect the expertise teachers bring with them as they engage in learning about practice.

To conclude this introduction, I share two important notes that will support the reading of this dissertation. First, chapters two, three, and four each represent three separate manuscripts from a larger study. Though each chapter represents significant ideas in its own right and were written to stand alone, they share a common review of the literature, learning theory, and conception of practice. Second, though these chapters were written as independent, the tables and figures throughout this dissertation are numbered as they appear according to the whole dissertation (see the list of tables and figures on pages viii and ix).

# CHAPTER II

# CONCEPTUALIZING PRACTICE FOR TEACHER LEARNING IN PROFESSIONAL DEVELOPMENT

Over the past decade, teacher educators have answered Grossman, Hammerness, and McDonald's (2009) call to identify core practices of teaching that are responsive to, and supportive of, student learning. Concurrently, they have explored pedagogies to assist prospective teachers in understanding the aims of these practices and learning to enact them. A focus on core practices and pedagogies to support their enactment have been shown to help prepare novices to begin the complex work of teaching (Campbell & Elliot, 2015; Ghousseini & Herbst, 2014; Han & Paine, 2010; Hunter & Anthony, 2012; Kazemi, Ghousseini, Cunard, & Turrou, 2016; Kazemi & Wæge, 2015; Lampert et al., 2013). While interest in core practices has grown, scholars have noted that the lack of a robust and shared conceptualization of practice impedes design and research of teacher (Forzani, 2014; Jacobs & Spangler, 2017; Jansen, Grossman, & Westbroek, 2015; McDonald, Kazemi, & Kavanagh, 2013), and that a shared model of practice might make progress in accumulating knowledge and building theory of teacher learning of practice (Clark & Hollingsworth, 2002; Opfer & Pedder, 2012).

In this paper, I use hierarchical modularity (Simon, 1973, 1996), a theory to manage complex systems, to conceptualize teaching to address two problems. I show how this model can address a set of design considerations (Jacobs & Spangler, 2017) and learning tensions (Jansen, Grossman, & Westbroek, 2015) of a core practice approach and investigate its potential for designing and researching teacher learning of core practices in professional development. My argument for this conceptual model of practice, and the examples I share to investigate it, reside within the context of a 2-year design-research study of a professional development for secondary mathematics teachers focused on enacting core practices of ambitious teaching. Recognizing that historically, learning opportunities for teachers have been focused on the acquisition and appropriation of knowledge (Goldsmith, Doerr, & Lewis, 2014; Grossman, Smagorinsky, & Valencia, 1999), our research group was interested in designing for and researching teacher learning in professional development where practice is the primary focus. Thus, we drew upon the literature on core practices and a teacher educator pedagogy, rehearsal (e.g. Lampert et al., 2013), to design our professional development and used hierarchical modularity to conceptualize practice for our design and research.

In what follows, I first describe a core practice approach and two sets of challenges of using this approach to design for and research teacher learning in professional development. Next, I introduce hierarchical modularity (Simon, 1973) as a perspective for managing and researching complex systems. Then, I outline the professional development and how hierarchical modularity supported our design efforts and attended to the challenges of a core practice approach put forth by Jacobs & Spangler's (2017). Finally, I focus on three tensions in researching teacher learning of core practices (Jansen et al., 2015) and share examples from teachers' rehearsals in professional development and their lessons during the school year to illustrate how hierarchical modularity enabled research on teacher learning of practice. I conclude this paper by highlighting important implications of using hierarchical modularity for teachers, teacher educators, and researchers.

## Challenges of a Core Practices Approach for Teacher Learning

Over the past several decades, researchers have made significant advances toward illuminating the kind of mathematics instruction we wish to see in classrooms. Recently, scholars have organized these efforts around what is described as *ambitious teaching.* Broadly, ambitious teaching,

> requires that teachers teach in response to what students do as they engage in problem solving performances, all while holding students accountable to learning goals of the discipline that include procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. (Kazemi, Franke, Lampert, 2009, p. 1)

Ambitious teaching requires teachers to be proactive and intentional in supporting students by problematizing existing ideas, eliciting and responding to students' individual and collective mathematical thinking, and scaffolding classroom discussions toward formalized learning goals for students (Kazemi, Franke, Lampert, 2009; Munter et al., 2015; National Research Council, 2001; Smith & Stein, 2011). Such instruction has been shown to have positive implications for student learning (Boaler & Staples, 2008; Franke, Webb, Chan, Ing, & Battey, 2009; Stigler & Hiebert, 2004; Tarr, Reys, Reys, Chavez, Shih, Osterlind, 2008).

Researchers and teacher educators have worked to unpack ambitious teaching in different ways to ensure that this broad vision is further specified to support content-specific teacher learning, such as ambitious science teaching (Thompson, Windschitl, & Braatan, 2013), ambitious mathematics teaching (Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010), and ambitious history teaching (Grant & Gradwell, 2009). As a part of unpacking ambitious teaching, teacher educators have worked to identify essential or "core" practices of teaching that occur frequently in teaching, can be enacted using different instructional approaches, allow teachers to learn more about students and the work of teaching, preserve the complexity and integrity of teaching, and are research-based and have impacts on student learning (Grossman et al., 2009).

While identifying core practices, teacher educators have also drawn from Grossman and colleagues (2009) influential study of the preparation of individuals for relational professions to address critiques that teacher learning of teaching has been too focused on the knowledge needed to teach, with less attention on how to use this knowledge in practice (Ball & Forzani, 2009; Forzani, 2014; Grossman et al., 2009). These authors call for learning opportunities that provide teachers with occasions to engage in representations, decompositions, and approximations of the practices of teaching. Representations of practice are the ways in which teachers engage with images of teaching made visible by teacher educators (e.g. through model lessons, video cases of teaching). Decompositions of practice are the ways in which teachers engage in deconstructing practice to highlight and specify particular aspects of teaching (e.g. launching a task, noticing, facilitating a mathematics discussion). Approximations of practice refer to learning opportunities for teachers to engage with and try out practices of teaching (e.g. analyzing student work, rehearsing a practice).

Teacher educators often engage prospective teachers in cycles of investigation and enactment to organize learning of ambitious teaching using these three concepts. During these cycles, prospective teachers observe a representation of ambitious teaching, collectively analyze a lesson or aspects of a lesson to decompose practice, and prepare and rehearse individual or sets of practices with their peers or students while receiving feedback from teacher educators (Lampert et al., 2013). Thus, the goal of a core practice approach is to “focus on both the core practices of teaching and on the pedagogies of teacher education used to prepare novices to enact these practices in ways that are responsive to the unique needs of their K-12 students” (Grossman, 2018, p. 2). Emerging research on the use of rehearsal and a core practice approach to teacher learning has demonstrated its value in assisting prospective teachers in understanding and learning to enact ambitious teaching practices (Boerst et al., 2011; Campbell & Elliot, 2015; Hunter & Anthony, 2012; Lampert et al., 2013; Tyminski, Zambak, Drake, & Land, 2014; Ghousseini & Herbst, 2014; Han & Paine, 2010; Kazemi et al., 2016).

**Design Considerations for Teacher Learning of Core Practices**

While teacher educators have found a core practice approach to be a productive way to design for and investigate teacher learning, Jacobs & Spangler (2017) highlight four challenges to this approach. First, teacher educators must choose the appropriate grain size of practices to focus on relative to their learning goals and contexts, and the unit of instructional time of focus. Because variations in the referent for “size” are often implicit or underspecified, the literature is inconsistent and provides little guidance for the choices designers must make. For example, launching an activity (Jackson et al.,

2013) or facilitating a whole-class discussion (Stein et al., 2008) can be seen as large grain-size practices, while pressing a student to justify their reasoning (Kazemi & Stipek, 2001) or orienting students to one another's ideas (McDonald, Kazemi, & Kavanagh, 2013) can be seen as smaller grain-size practices. Focusing on too small of a grain size may disrupt teacher learning of the practice in relation to larger practices in which they may be nested, while too large of a grain size may require further unpacking to be useful for design and teacher learning.

A second challenge is identifying which practices to focus on and ensuring that in learning those practices, teacher educators meet their multiple goals for teacher learning (Jacobs & Spangler, 2017). Similarly, a third challenge, is the need to attend to the relational nature between and across practices, as "individual core practices do not occur in isolation but rather in the context of other practices" (p. 768). For example, the small grain-size practice of orienting students to each other's thinking could be nested within a larger grain-size practice of eliciting and responding to students' thinking, which could in turn be nested within an even larger grain-size practice of facilitating whole-class conversations around content and students' thinking. Thus, an important design decision in choosing which practices to focus on, is attention to both the complexity of teaching and any potential nested or connected relationships between practices of varied grain sizes.

Fourth, because a core practice approach to supporting teacher learning has become more prevalent, there is a need to develop a common and precise "technical language" to describe practices (Jacobs & Spangler, 2017). To develop a common language around practices, a group of teacher educators and researchers have come together as the Core Practice Consortium to clarify how practices are identified and defined. This group has refined definitions of practices that are common across content domains yet adaptable for content-specific settings, such as identifying the practice of leading a whole class discussion (Grossman, 2018). After providing a broad definition for the practice, these teacher educators provide expanded definitions when describing it with attention to teaching history, language arts, mathematics, and science.

Though the Consortium has made progress in developing a common language for practices, Jacobs & Spangler (2017) highlight that a common language is also needed for the instructional aim for enacting practices of different grain sizes. For example, one might consider whether the field should call the aim for facilitating a discussion its *goal* while also calling the aim for eliciting students' thinking as its *goal*; or whether the field should differentiate the naming of these aims by calling one its *purpose* and the other its *goal* to characterize aims of different grain sizes. The important point here is that not only do we need a common language for the practices themselves, but also for the reasons behind enacting them. Clarifying aims of different grain-size practices could improve the design of teacher educator pedagogies for enacting practices and facilitate the meanings teachers make of the complexity of nested and connected practices and myriad of aims they attend to during instruction.

These considerations highlight challenges in using a core practice approach for design and the framing of research questions, and motivate the need for a conceptual model that can address them. Designers choosing to use a core practice approach must attend to these challenges while balancing broader goals for teacher learning and where teachers are in their professional trajectories (Jacobs & Spangler, 2017).

**Tensions in Teacher Learning of Core Practices**

In addition to these design considerations, Jansen, Grossman, and Westbroek (2015) highlight three "unresolved issues" or tensions in design and research efforts of a core practice approach. They identify and explain these tensions in the context of teacher preparation. I extend each of these tensions to include considerations for teachers and their existing systems of practice to focus on efforts to design and research teacher learning of practice in professional development.

First, a core practice approach has predominantly focused on decomposing ambitious teaching into practices, with little attention to its "complement" of *recomposing* practices to support teaching broader routines or whole lessons (Jansen et al., 2015). While learning to enact individual practices is important, a failure to attend to how multiple practices are brought together may hinder prospective teachers' learning and their ability to enact multiple practices to meet learning goals. In addition, their minimal experience requires that they must learn both the practices themselves as well as the aim for enacting them. For teachers, who already have a system of teaching that includes many practices, this tension brings into focus a need to not only decompose ambitious teaching, but also to support teachers in decomposing their own teaching to examine their practices and the ways they could bring them together to be more responsive to students. Thus, this tension provides a lens for examining teacher learning of core practices as teachers work to recompose their practice over time as they learn to teach more ambitiously.

Second, Jansen et al. (2017) noted a core practice approach for prospective teachers has focused heavily on the skills of teaching, with little consideration of the ways teachers develop the will or motivation to teach ambitiously (Jansen et al., 2015). They call on designers to attend to both the skillful enactment of practices and the development of dispositions and aims necessary for their enactment. For teachers, this tension requires designing for learning that respects teachers' existing practices and their aims for enacting them, while providing a conception of practice that teachers can weigh against their current practice to envision ways they can teach more ambitiously.

Third, tension exists between the development of routines of practice and the development of adaptive expertise (Jansen et al., 2015). As prospective teachers develop routines for practice they can use in their beginning years of teaching, they also need to develop the improvisational skills necessary to adapt these routines in response to issues that arise during instruction. For teacher educators working in professional development, they must also attend to the fact that teachers already have existing routines of practice, and in some instances are adaptive experts in responding to students during instruction to support learning.

Together, the design considerations (Jacobs & Spangler, 2017) and learning tensions (Jansen et al., 2015) warrant an exploration of a conceptual model that can attend to and reconcile them for teachers with existing systems of practice. These considerations and tensions have predominantly been hypothesized within the realm of teacher preparation and I have briefly addressed similar issues for teachers with existing systems of practice. I know turn to hierarchical modularity (Simon, 1973) as a way to conceptualize teaching to manage these challenges, design for teacher learning of core practices, and frame research on teacher learning.

## Hierarchical Modularity

To promote a common framework for understanding and researching complex systems, Simon (1973) introduced hierarchical modularity as a way to theorize complex systems for learning and research. He posited that complex systems are "nearly decomposable" as a collection of "localized subsystems", with properties that are both specific to each subsystem and relate to other subsystems within the larger system. He contended that most systems could be classified as a complex hierarchical structure, regardless of "whether those systems are physical, chemical, biological, social, or artificial" (p. 3) and that modularity could be useful in modeling social phenomena. A hierarchical modular approach to understanding, adapting, and problem-solving within complex systems has proliferated throughout a broad range of research fields, including organizational management (Sanchez & Mahoney, 1996), biology (Kashtan & Alon, 2005), engineering design (Sosa, Eppinger, & Rowles, 2007), clinical psychology (Chorpita, Daleiden, & Weisz, 2005), and cultural change (Wimsatt, 2013).

As one example, Chorpita and colleagues (2005) used a modular approach to conceptualize a protocol for psychologists to use in therapeutic settings. An aim of their work was to explore a model for treatment design that could be "applied across multiple theoretical orientations" (p. 141). Using several examples, they highlight how their proposed model promotes efficiency in both design and intervention, attends to greater complexity and variation than other protocols, and preserves the integrity of the practices of psychology. In addition, they show how modularity allows for “rapid adaptation” of practice and shared preliminary data of both novice, “graduate trainees” and professional community therapists satisfaction with a modular approach to therapy procedures. They conclude by noting the limits of modularity in addressing the social aspects of therapeutic relationships, yet its potential in supporting the field of clinical psychology in addressing design and research challenges in their field.

More recently, Jansen and colleagues (2015) introduced modularity as a potential way to address some of the challenges of a core practice approach. However, these authors focused their efforts on prospective teacher education and did not provide empirical support for their claims, instead introducing anecdotal scenarios where it may be useful. In this paper, I extend their ideas to include attention to teacher learning in professional development and provide further empirical support for this model.

Researchers describe hierarchical modular systems from both a structural and functional perspective to highlight the observable characteristics of a complex system and the subsystems embedded within (Simon, 1965, 1973; Bethel & Richardson, 2010). While there are many ways to visualize hierarchical modular systems, in Figure 1 I provide a framework to support an understanding of the way I envisioned modularity for design and research of teacher learning of practice in professional development.

| Complex System | | | | | Emergent Property |
|---|---|---|---|---|---|
| **Level 1** | **Subsystem 1.1** | | **Subsystem 1.2** | | Aim of subsystem 1 |
| **Level 2** | **Subsystem 2.1** | **Subsystem 2.2** | **Subsystem 2.3** | | Aim of subsystem 2 |
| **Level *n*** | **Subsystem *n*.1** | **Subsystem *n*.2** | **Subsystem *n*.3** | **Subsystem *n*.4** | Aim of subsystem 2 |

Figure 1. Hierarchical Modularity for a Complex System

Structurally, hierarchical modularity is simply a way to describe a complex system, identify subsystems at different "levels" within that system, and mark relationships between and across a system (Simon, 1965, 1973; Bethel & Richardson, 2010). Functionally, hierarchical modularity describes the effects that subsystems have on and across different parts of a system and the "emergent" properties or aims that can be inferred when subsystems are made visible. It is important to note that the number of subsystems at each level of the system and any potential relationships among subsystems, both on and across different levels, is contingent upon the complex system being modeled.

Simon (1965, 1996) describes three characteristics that are central to understanding hierarchical modularity and its utility in managing complex systems. First, strong connections exist within an individual subsystem (e.g., Subsystem 2.2), what Jansen and colleagues (2015) refer to as "internal coupling". Second, "horizontal coupling," describes that certain connections exist across subsystems at the same level (e.g., Level 1) that do not exist at other levels. Third, "vertical coupling" depicts that within a subsystem at a higher level (e.g., Subsystem 1.1), lower level subsystems (e.g., Subsystem 2.2) that are a contained within a larger subsystem can also take place in other larger subsystems.

These characteristics have three implications for both design and research of complex systems. First, drawing upon the characteristic of “internal coupling” one could focus on the properties of an individual subsystem while temporarily suspending attention to other subsystems of the complex system. Second, one could leverage the “horizontal coupling” characteristic to focus on a specific level of the system. Third, drawing upon “vertical coupling” one could investigate the ways that lower level subsystems vertically relate to multiple subsystems at higher levels. Simon (1973) argues that attention to both the structural and functional perspectives and these characteristics of complex systems supports the overall management of the complexity of a system, the analysis of evolution and change within a system, and allows for better understanding of the ways in which small changes at lower levels of the system spread to produce large changes within upper levels of the system.

I began by highlighting two sets of considerations when using a core practice approach for designing and researching teacher learning of ambitious teaching. First, teacher educators must determine the appropriate grain sizes of practice and which practices to focus on, attend to the relational nature between and across practices, and specify language to describe practices and their aims (Jacobs & Spangler, 2017). Second, when researching teacher learning of core practices, researchers must attend to: the relationship between decomposing teaching into practices and recomposing practices back together toward whole lessons; balancing the development of the skills of teaching and teachers' aims for enacting teaching; and the development of routines for practices and the adaptive expertise needed to ensure that these routines are used in service of and in response to students (Jansen et al., 2015). Using these two sets of considerations, I now use hierarchical modularity to describe how it can address the design considerations and learning tensions of a core practice approach for teacher learning of ambitious teaching. To do so, I first describe how hierarchical modularity supported our efforts to design a practice-based professional development and addressed Jacobs & Spangler's (2017) design considerations. Next, I highlight the ways hierarchical modularity can support research of teacher learning and address the learning tensions of a core practice approach described by Jansen and colleagues (2015). To do so, I provide specific examples from our professional development to illustrate how hierarchical modularity can support research on teacher learning of practice.

## Designing PD that Respects and Challenges Teachers' Practice

Researchers have argued that professional development should be intensive and ongoing; connected to content, practice, and students' thinking; and encourage shared participation (Darling-Hammond et al. 2009; Desimone, 2009; Elmore 2002; Heck et al. 2008; Sztajn et al., 2007; Yoon et al. 2007). Many studies have reported that professional development of this form supports changes in teachers' practice, including their use of students' mathematical thinking in instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Wilson, Sztajn, Edgington, & Myers, 2015); curricular materials (Tarr, Reys, Reys, Chavez, Shih,

& Osterlind, 2008); and rich mathematics tasks (Boston & Smith, 2009; Stein, Grover, & Henningsen, 1996), among others.

While focusing on these design features of professional development and broadly connecting design to practice is needed, teachers do not come to professional development to simply engage with new ideas about practice. Rather, they engage in ideas that may be different from those that have guided their practice in the past and bring with them their existing systems of practice and ways of reconciling the competing challenges and complexities of teaching (Kennedy, 2016). For teachers to use their learning from professional development in their practice, new ideas introduced must be reconciled with teachers' existing systems of practice.

More than being "practice-based", professional learning tasks should seek to relate new learning to teachers' existing practice, and doing so has been shown to influence teachers' instructional strategies and lead to changes in practice that can be seen as more ambitious (Darling-Hammond et al., 2009; Desimone, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001; Goldsmith et al., 2014; Hill & Cohen, 2001). Because teachers' practice is a primary site for teacher learning in professional development (Ball & Cohen, 1999; Sztajn, Borko, & Smith, 2017), professional learning tasks should both problematize aspects of teachers' practice and provide opportunities for inquiry and experimentation (Cochran-Smith & Lytle, 1999; Franke, Carpenter, Levi, & Fennema, 2001; Kazemi & Hubbard, 2008; Lampert & Ball, 1998).

Similar to the work in prospective teacher education, designers of professional development have increasingly engaged teachers in representations, decompositions, and approximations of teaching to provide opportunities for exploring teaching and conjecturing new possibilities (Ball & Cohen, 1999; Wilson & Bern, 1999, Silver et al., 2007). Structured around various artifacts of teaching such as student's written work (Kazemi & Franke, 2004), classroom video (van Es & Sherrin, 2006), or clinical interviews with students (Jacobs & Empson, 2015; Wilson, Mojica, & Confrey, 2013), teacher educators use these artifacts of practice as part of professional learning tasks to make practices of teaching public for examination (Ball & Cohen, 1999; Silver, Clark, Ghousseini, Charolambous, & Sealy, 2007; Smith, 2001). Careful attention to the design of learning tasks has potential to improve teacher learning (Swan, 2007; Silverman & Thompson, 2008), and some teacher educators encourage approximations that provide teachers with deliberate practice closely aligned with in-the-moment complexities of teaching, including rehearsals (Ball & Cohen, 1999; Grossman & McDonald, 2008; McDonald, Kazemi, & Kavanagh, 2013; Sandoval, Kawasaki, Cournoyer, & Rodriguez, 2016).

Though most efforts around core practices have been conceptualized, designed, and explored for the purpose of preparing prospective teachers as they begin their teaching careers, our research team was interested in the potential of a core practice approach and how the use of rehearsals might support secondary mathematics teachers in their enactments of ambitious teaching. Throughout our design and research, we have begun to build a case for this approach (Webb, in preparation a; Webb, in preparation b; Webb, Wilson, Martin, & Duggan, 2015).

We have found that using hierarchical modularity to conceptualize practice supports efforts to design professional development focused on practice and research teacher learning of practice. I now describe the professional develop that is the context of this work and highlight the ways modularity addressed Jacobs & Spangler's (2017) design challenges of a core practice approach. Following this, I describe how hierarchical modularity can be used to address Jansen and colleagues (2015) tensions and share examples to illustrate how hierarchical modularity supports research on teacher learning of practice.

**Context: Practice-Based Professional Development**

The professional development project used as context in this paper took place over two implementations of a practice-based professional development with secondary mathematics teachers. The project was focused on core practices of ambitious mathematics teaching and organized around cycles of investigating core practices by engaging with representations, decomposing practice, and approximating core practices of ambitious teaching in rehearsal. In addition, the project was also designed around mathematics content central to secondary mathematics. Each implementation was built from a consensus view for effective professional development (Darling-Hammond et al. 2009; Desimone, 2009; Elmore 2002; Heck et al. 2008; Sztajn et al., 2007; Yoon et al., 2007) and designed for a 12-month period consisting of a 60-hour summer institute followed by approximately 20-hours of follow-up meetings throughout the school year.

For each of the summer institutes, we developed sequences of professional learning tasks (Wilson, Sztajn, & Edgington, 2013) focused on representing ambitious teaching, decomposing ambitious teaching to make the core practices of focus salient for analysis and discussion, and approximating core practices in practice-based ways such as analyzing student work or classroom videos. After engaging in approximations of practice, each teacher rehearsed three large-grain size core practices: launching a mathematics task, monitoring small group engagement, and facilitating whole class discussions. During all rehearsals, a teacher educator served as facilitator, stopping the rehearsal at various times to elicit reasons for the decisions a teacher made or her or his conjecture about future actions they could take in the rehearsal.

During each summer institute, as teachers engaged with the mathematical ideas of focus in the professional development, we shared research-based knowledge of students' mathematical thinking in these domains. In doing so, we aimed to support teachers in building upon these ideas to leverage students' thinking as they engaged in rehearsals of core practices. Throughout the year, teachers met bi-monthly with the research team after school to relate their work from the summer institute to their teaching during the school year by analyzing their classroom videos, planning for instruction, or rehearsing core practices for upcoming lessons.

From the outset of the professional development, participating teachers had a clear learning goal of learning to teach more ambitiously. As researchers, we had a goal of understanding how teachers came to learn to enact core practices of ambitious teaching and determining if rehearsals of core practices were useful in supporting their learning. Therefore, our design challenge was to conceptualize teaching in a way that acknowledged and respected teachers' existing practices while creating opportunities to experiment with new or modified practices that are more ambitious. I now provide examples of how hierarchical modularity resolved Jacobs & Spangler's (2017) four design considerations for using a core practice approach that we attended to in our design.

**Conceptualizing Ambitious Teaching for Teacher Learning**

At the outset of our efforts, we aimed to conceptualize teaching in a way that held to our commitments of respecting teachers' existing practice, while also providing ways to problematize their existing conceptions toward a vision of teaching that may be more ambitious. We conjectured that using a core practice approach and rehearsals as a part of cycles of investigating core practices could support teachers in situating the practices within their existing system of teaching, problematizing their past enactments, providing them with opportunities to engage in imaginative practice in rehearsals, and enacting these practices in their classroom teaching. The structural perspective of hierarchical modularity provided a way to organize our model for practice.

**Choosing practices and attending to grain size.** Because we had a broad goal of supporting teachers in teaching whole lessons that had more ambitious goals for student learning, we chose to strategically target three, large grain-size practices with a few supporting smaller grain-size practices. We focused on the larger practices of *launching* a mathematics task (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013), *monitoring* students engagement in the task (Kazemi & Hubbard, 2008), and *discussing* the task with the whole class toward a mathematical goal (Stein, Engle, Smith, & Hughes, 2008) for two reasons. First, each of these practices have been sites for individual research and have been shown to be productive practices to support student learning (Jackson et al., 2013; Kazemi & Hubbard, 2008; Stein et al., 2008). Second, we conjectured that these practices were a part of teacher's current systems of practice already.

In addition to these larger practices, we chose a set of smaller practices that could be nested within and used across each of the larger practices throughout a lesson. We select smaller practices that teachers typically used in instruction and a few that we conjectured would support teachers in enacting more ambitious lessons that leveraged and built upon students' mathematical thinking. The five smaller practices we chose were *probing* students thinking, *revoicing* a students' contribution, *explaining* a mathematical idea or contextual feature of the activity, *pressing* students' reasoning, and *orienting* students to one another's ideas. We conjectured that probing, revoicing, and explaining were common practices that teachers enacted throughout their lessons, while pressing and orienting may be less common given their responsive nature. That is, enacting the practice of pressing or orienting requires that teachers have already engaged with students around content to develop goals for enacting pressing or orienting. Pressing students' reasoning (Kazemi & Stipek, 2001) and orienting students to one another's mathematical ideas (Boaler & Staples, 2008; McDonald, Kazemi, & Kavanagh, 2013) have been shown to support students in advancing their mathematical thinking toward disciplined forms of mathematics and teachers in more deeply understanding their students' reasoning.

**Relating practices and developing a common language.** While the structural perspective of hierarchical modularity was supportive of our efforts to choose practices of varied grain sizes to focus on, we also needed a way to manage the relational nature between practices of different grain sizes and a common language to discuss the aims for enacting these practices ambitiously. The challenge we faced was that while choosing and discussing these practices was important, what was more important were teachers aims for enacting these practices. Because teachers already enact the majority of these practices, our design needed to include opportunities to represent, decompose, and approximate them with more ambitious aims. To do so, we drew upon the functional perspective of modularity to further define the aims of each practice.

A tension in design is that some teacher educators have described “actions meant to facilitate learning typically through a combination of speech and gesture” (Harris, Phillips, & Penuel, 2012, p. 776) as both a move and a core practice (Grossman, 2018), while others have referred to them as instructional moves (Harris et al., 2012). For our design, the functional perspective of hierarchical modularity provided a way to organize the aims for enacting core practices of different grain sizes as well as make decisions regarding the naming of these smaller practices. Following hierarchical modularity, we chose to label the smaller core practices as *instructional moves* made visible during instruction at lower levels of the system. This decision provided a way to discuss these smaller practices as nested and connected within and across the larger practices and provided a stable language in our model.

Lastly, we sought a way to manage the technical language (Jacobs & Spangler, 2017) needed to describe the aims for enacting practices at different levels. We chose to describe aims across different levels of the system using *vision* for broad aims for ambitious teaching, *purpose* for aims for enacting larger practices, and *goals* to describe the aims for enacting the smaller practices or instructional moves. Outlined in Figure 2 is a summary representation of the way we used hierarchical modularity to conceptualize teaching for the design of our professional development.

| | | | | |
|---|---|---|---|---|
| **System** | **Ambitious Mathematics Teaching** | | | *Vision* for mathematics teaching |
| **Core Practices** | **Launching** | **Monitoring** | **Discussing** | *Purpose* for each practice |
| **Instructional Moves** | **Probing Revoicing Explaining Pressing Orienting** | | | *Goal*(s) for moves |

Figure 2. Hierarchical Modularity of Ambitious Mathematics Teaching

**Summary of the Summer Institute**

For the summer institute, we used this model and engaged teachers in sequences of tasks that represented and decomposed practice to promote both the core practices and instructional moves. For launching, monitoring, and discussing, we formalized these practices into frameworks (Appendix A) that highlighted a broad purpose of the practice and possible goals teachers could have for their instructional moves. Teachers used these frameworks when approximating the larger practices – first using artifacts of practice and then by rehearsing.

## Researching Teacher Learning

I have demonstrated how a conceptual model of teaching built from a hierarchical modular perspective (Simon, 1973) addressed Jacobs & Spangler's (2017) design considerations and supported the design of our professional development. I now turn to how our use of hierarchical modularity to research teacher learning of core practices resolved Jansen and colleagues' (2015) three tensions. Using examples from four teachers' rehearsals of core practices and lessons over two years, I illustrate how hierarchical modularity enabled research on teacher learning of practice.

**Recomposing Core Practices of Ambitious Teaching**

As researchers interested in the ways teachers bring practices together over time, attending to the vertical and horizontal coupling characteristics of hierarchical modularity supported analyses of teacher learning of practice. Elsewhere, I have reported on the ways teachers' enactments of individual practices evolved over time and how vertical coupling helped facilitate an understanding of the ways teachers recomposed their enactments of larger practices to include instructional moves that were responsive to and supportive of students' thinking (Webb, in preparation b). In addition, I have also reported on the ways teachers' recompositions of practices related to their engagement in rehearsals in professional development (Webb, in preparation a). Here, I use horizontal coupling to provide an example of how one teacher, Sara, recomposed her practice over time to bring together launching, monitoring, and discussing to enact whole lessons that progressed to be more ambitious. In this example, I move back and forth between Sara's enactments of these practices and her participation in rehearsals in the two summer institutes to highlight the ways her engagement in rehearsals related to the ways she brought together practices in her classroom teaching. A fuller explanation of these enactments is reported elsewhere (Webb, in preparation b).

**Lesson #1, #2, and #3.** Prior to Sara's participation in the professional development, we modeled a series of ambitious mathematics lessons to provide teachers with a representation of what ambitious mathematics teaching could look like. We then asked teachers to implement one of these lessons with students. In her first lesson, Sara launched the task by reading the problem aloud, stating, “I am not going to answer any questions at this point,” and had students begin to work on the task. Students spent the entirety of the class period engaging with the task in small groups and Sara’s purpose for monitoring was to support students in engaging with the task toward their own ideas. To meet this purpose, she predominately used a pattern of probing students to understand their thinking and mathematical work, and then explaining contextual features of the task and mathematical procedures so that students could continue to engage. Her lesson concluded when the period ended, and Sara did not get to a whole class discussion.

After the first implementation of the summer institute, in Sara’s second and third lessons, she recomposed her enactment of the practice of launching to provide opportunities for students to understand both the context and goal of the task. Sara did not use pressing and orienting moves during these launches. In both of these lessons, as she monitored student’s progress with the task, she supported students in engaging more deeply with the mathematics of the task, ensured that they engaged with each other’s mathematical thinking, and used pressing and orienting moves to progress students toward her mathematical goal. In both of these enactments, she again did not have a whole class discussion with students about the mathematics of the task.

Sara’s recompositions of the large and small grain-size core practices across the first implementation of the professional development evolved to include a more ambitious purpose for launching, the addition of pressing and orienting moves within the practices of monitoring, and did not include discussions. Using this year-one recomposition, I now go back into the first summer to briefly provide evidence of ways in which Sara's rehearsals of the three larger practices may have related to her recomposition.

**Summer Institute #1.** In Sara's launching rehearsal, she rehearsed launching toward the purpose promoted in launching framework (Appendix A) and noted her prior difficulty in launching activities in ways that were responsive to students, stating,

> what I am learning is that I want someone to tell me, "yeah do that every time". But that can't be given, every situation is different. That is what I have to come to grips with. You have to analyze your purpose and where you want to go.

In her monitoring rehearsal, Sara almost solely used probing moves and noted that pressing and orienting moves were "not the type of questions I have used in my classroom." In her discussion rehearsal, she was able sequence a set of student approaches, predominantly used probing moves, yet noted that,

> there are so many different ways you can go and there are so many factors to consider...[facilitating discussions] is so much more difficult, so much more intense...I think just ... continu[ing] to grow as a teacher from year to year and not try and bit it all off at once.

Across these rehearsals, Sara made meaning of the practices and was able to try out enacting imaginative practices that were more ambitious. In doing so, I conjecture this relates to the horizontal recomposition of her enactments across her first three lessons. By this I mean that as she added the core practice of launching, it facilitated greater opportunities to support students' productive engagement during monitoring, and for Sara, she was not ready to recompose her lesson to include discussions as she noted in her comments during her discussion rehearsal.

**Lesson #4 and #5.** After the second summer institute, in Sara's fourth and fifth lessons her enactment of the practice of launching incorporated both orienting and pressing moves, and her enactment of the practice of monitoring included an increased use and/or quality of pressing and orienting moves. In these two lessons, Sara had whole class discussions with students about the mathematics of the task, and in these discussions, she sequenced students approaches and used pressing and orienting moves to progress toward her mathematical goal for the lesson. I now go back into the second summer institute to briefly provide evidence of ways Sara's engagement in rehearsing the three larger practices may have supported her ability to enact whole lessons that included discussions.

**Summer Institute #2.** In Sara's launching rehearsal, she used orienting moves and was more responsive to students' thinking, stating that she now had,

> an opportunity to practice handling students' responses on the fly in the moment and [it] helped me realize how every decision I make effects something...I feel like I have a much better picture of what a launch should be and know where I am going...In doing this [the rehearsal] I am getting to the point where I am beginning to think I can do this [enacting whole lessons].

In her monitoring rehearsal, Sara used orienting and pressing moves to ensure that students were working together, noting that,

> in the past, I would just monitor to check for understanding...but I never had the drive to get to a certain [mathematical] goal in terms of more understanding...by

> practicing [rehearsing] I am constantly thinking about...where are we going with this...to reach the purpose.

Finally, in her discussion rehearsal she stated,

> It's like its better the second time around because you have more [knowledge and experience] to help you through it. I think if I can do it three or four more times I will be really good at it.

In these rehearsals, she was able to again try out the practices toward ambitious purposes and found the rehearsal as a space to gain confidence for future enactments. In doing so, she continued to find rehearsing generative and tried out using orienting moves with goals that both aligned with the frameworks and were focused on a mathematical goal for a lesson. Across these rehearsals, Sara was able to continue to make meaning of the practices and try out enacting imaginative practices in ways that were more ambitious. In doing so, I conjecture this relates to the horizontal recomposition of her enactments in her fourth and fifth lessons that included a refined purpose for the practices of launching and monitoring and the addition of the practice of facilitating discussions.

While this example of Sara's recompositions of these practices over two years is concise and lacks specificity, it highlights the ways a hierarchical modular approach could facilitate researchers understanding of how teachers' enactments of practices work together across whole lessons and changes in enactments of individual practices relate vertically and horizontally within and across a lesson. By looking back and forth between teachers' classroom teaching and participation in rehearsals over time, I see hierarchical modularity as a way to facilitate an understanding of the coevolution of teachers' participation in professional development and their classroom instruction (Clark & Hollingsworth, 2002; Kazemi & Hubbard, 2008; Sztajn et al., 2017).

## Understanding the Coevolution of Shifts in Teachers Skill and Aim

To research the relationship between changes in teachers' enactments of practices, and their aims for enacting them, I draw upon the structural and functional perspectives of hierarchical modularity. This analysis was supported by a conceptualization of practice that differentiated between aims at different levels of the system (i.e., vision, purpose, goal in Figure 2). Elsewhere, I have highlighted how vertical coupling supported an understanding of the ways changes in teachers' goals for enacting pressing and orienting moves related to changes in the overall quality of their enactments of the larger practices of launching, monitoring, and discussing (Webb, in preparation b). Here, I provide an example of the way these co-occurring shifts in skill and aims at different levels of the system could also relate to changes in a teachers' vision for mathematics teaching.

**Brenda's initial skill and aim.** As an example, I draw on the coevolution of skill and aim of one teacher, Brenda. In analyzing Brenda's first lesson that we asked her to teach, Brenda's *vision* for teaching could best be described as providing students with opportunities to experience "real world" mathematics by engaging in cognitively demanding tasks to develop their own ideas about the mathematics. Moving down one level to the three larger practices, Brenda's purpose for launching aligned with her overall vision as she allowed students to develop their own understanding of the context and mathematical goal of the task. Similarly, her purpose for monitoring aligned with her vision as she allowed students to continually engaging with their own ideas, and often withheld support when students encountered difficulties with the task. Finally, her purpose for the whole class discussion also aligned with her vision as she focused on eliciting answers from students, evaluating their correctness, and did not attend to students' understanding of the mathematics. Additionally, across this lesson, Brenda did not use instructional moves that required attention and response to students' mathematical thinking (i.e. pressing and orienting) and her goals for the instructional moves she used (probing and explaining) aligned with her purpose for enacting the larger practices and her broad vision to provide students with "discovery" learning opportunities.

**Brenda's evolving skill and aim.** Throughout her remaining lessons over the two years of her participation in the professional development, Brenda's vision remained unchanged. However, moving down one level, Brenda's purpose for launching shifted to support students in understanding the task and over time grew to also provide opportunities for students to elaborate their thinking before engaging in the task. Brenda's purpose for monitoring remained unchanged until her last two lessons, where it evolved to support students in understanding the mathematics of the task in ways that progressed toward her goal for the lesson. Brenda's purpose for facilitating discussions shifted from eliciting student's answers to eliciting their mathematical thinking so she could focus on procedures related to her learning goal. Across Brenda's shifts in each of these practices, her goals for the moves she used related to these shifts in purpose in different ways. For example, her launches evolved to align to the purpose promoted in the professional development, but she did not use moves that were attentive or responsive to students' thinking (i.e. pressing and orienting). Her monitoring advanced to add the use of pressing and orienting moves with goals that were more responsive to students' mathematical thinking and her discussions progressed to support students in working toward her mathematical goal for the lesson.

To research teacher learning of practice, understanding the ways in which teacher's vision, purposes, and goals work together to relate to shifts in their practice is complex. Using hierarchical modularity provided a way to better understand the relationship between teachers evolving skills and aims. To design for teacher learning, this conceptualization could also further support teacher educators in making decisions about how to foster individual teacher learning of ambitious teaching in ways that attend to aims at different levels. That is, while Brenda's goals for moves and purpose for practices progressed to be more ambitious, maintaining a less ambitious vision of teaching hindered her enactments of each core practice, even as these lower level practices changed. Thus, considering ways to respect and challenge teachers' existing aims at different levels of the system (i.e. vision, purposes, goals) to support their development of more ambitious practices and aims is essential. A hierarchical modular approach that attends to aims at different levels of the system can supporting teacher educators in adapting designed learning opportunities for teachers to attend to their individual shifts at these three levels and their own goals for learning.

**Developing Routines of Practice and Adaptive Expertise**

Respecting teachers existing system and routines of teaching, we designed opportunities in the professional development for teachers to engage with practices that they may already use toward more ambitious purposes. We also explored the ways teachers used their rehearsals and enactments of practices as sites for adapting our conceptualization of teaching over time to align with their own goals for teaching. I share two examples of the ways in which teachers engaged in adaptive practice in their classroom lessons and rehearsals in the professional development, and how hierarchical modularity could support further research efforts exploring these ideas.

**Adapting teaching in response to instructional decisions.** While some teachers, like Sara, progressively recomposed practices vertically (i.e. adding pressing and orienting moves) and horizontally (i.e. adding launching and discussing) over the course of their lessons, others came to the professional development with conceptions and enactments of practice that were more closely aligned with ambitious teaching and our conceptualization of practice promoted in the professional development. For example, across all five of Carla's lessons she almost always enacted each of the larger core practices, and within these practices varied in her use of pressing and orienting moves to support students. Her use of these moves did not progress over her lessons as with other teachers, rather they varied across her enactments of individual practices.

During one lesson, after Carla did not use pressing and orienting moves during her launch, she drew upon what we came to call "check-ins" to facilitate and progress her lesson toward her learning goal. In this lesson, she quickly launched the task and did not elicit students' understanding of potential barriers to engaging in the task, an explicit goal promoted in the framework for launching. As students began to work on the task, Carla spent time with several of the small groups addressing barriers, as students were unable to focus on the mathematical goal of the task. As this continued, rather than responding to other groups and expecting the same interactions, Carla chose to pause and "check-in" with the whole class to address the barrier so that students could continue engaging toward her goal for the lesson.

This instance happened prior to the second summer institute and the idea of "check-ins" resonated with teachers as the professional development progressed. In adding the idea of check-ins, these teachers engaged in adaptive practice as they worked to make meaning of our model and reconcile the practices promoted in the professional development with their existing systems of teaching. Over time, teachers proposed that we add check-ins to our model and we worked together to discuss the purpose of check-ins and its potential as a core practice. Attending to the ways teachers adapt their practice across their enactments and their participation in professional development could be a further site for understanding the meaning teachers are making of practice. In addition, respecting the adaptations teachers make to practice (e.g., addressing whether check-ins might be a mid-level practice between the practices we chose) could support teachers in their understanding and experimentation of bringing practices together toward whole lessons and researchers understanding of teacher learning of practice.

**Adapting design in response to teacher learning.** We were also interested in how teacher's engagement with our conceptual model over multiple years might be understood from a modular perspective. For example, throughout the first year teachers found facilitating discussions to be a useful practice to explore, rehearse, and enact. However, they found that in their classroom teaching, they struggled to transition between facilitating students' mathematical approaches, closing the lesson to clarify their goal, and formalizing students' procedural understanding of the mathematics. For the teachers who participated in both years of our summer institute, we brought them together to help us design the second summer institute to support both new and returning teachers. These teachers decided to add an additional practice, which we defined as *closing* a lesson to formalize a mathematical goal. In doing so, they chose to rehearse this practice in addition to the other three larger practices, thus structurally adding another practice to our model for ambitious teaching.

However, aiming to support the new teachers in the summer institute, we agreed that our original conceptual model was sufficient for these teachers to make meaning of and rehearse ambitious teaching practices. Additional research could explore how adding practices at the same level of a subsystem at various instances during professional development could further support teachers in enacting routines of practice and building adaptive expertise. These examples align with Jansen and colleagues (2015) conjecture that choosing to focus on specific core practices does not necessarily "constrain innovation, but rather that novices [and experienced teachers] can build on these foundational practices to experiment and innovate." (p. 144)

## Discussion

In this paper, I set out to examine how teaching might be conceptualized to design for teacher learning in ways that respect and challenge teachers' existing practice and support research on teacher learning. I began by summarizing a core practice approach and outlined a set of design considerations and learning tensions from the literature, focusing these challenges on teacher learning in professional development. I considered hierarchical modularity as a way to reconcile the challenges outlined, and provided an existence proof that this model could be a useful way to conceptualize teaching using a core practices approach for both design and research. To conclude, I highlight important considerations for researchers and designers who wish to use hierarchical modularity because they have implications for the ways we research and design for teaching learning of practice.

Hierarchical modularity is a rather simple concept. From a structural perspective, it broadly supports design by specifying the focus of practices promoted in professional development. From a functional perspective, a hierarchical modular approach facilitates an understanding of teacher learning of the complex system of ambitious teaching and the ways in which teachers recompose their practice, work to manage their aims for ambitious teaching at different grain sizes, and adapt practices to their own context and aims for ambitious teaching. While I found this theory useful for both design and research of teacher learning, I conclude by highlighting some limitations and considerations for future research.

First, one premise of using modularity to design and research teacher learning from a core practice approach is the need to attend to externalized action and make inferences about aims. Theoretically, modularity attends to this from the perspective of emergent properties, however, because this theory has traditionally been used in research on physical systems, one can easily see how inference would be more difficult when attending to social science design and research. As reform initiatives have outlined more ambitious expectations for teaching and efforts to design for professional learning have become more focused on the practices of teaching (Hiebert & Morris, 2012; Zeichner, 2012), scholars have argued for a need to bring together conceptual tools and theories of learning to attend to the complexities of practice and the multiple possibilities for pathways of changes in teachers' practice (Clark & Hollingsworth, 2002; Sztajn, Campbell, & Yoon, 2011). The conceptual model for practice I have introduced in this paper begins these efforts. In another paper, I combine this model with a complementary situated theory of learning (Wenger, 1998) to attend to individual learning of practice across settings (Webb, in preparation b). However, the question remains whether modularity can be brought together with other theories of learning and what affordances and constraints these theories might have on design and research.

Second, because this conceptualization focuses on externalized action, it fails to attend to less visible aspects of teaching, such as the ways teachers position students as learners (Wilson, Sztajn, Edgington, Webb, & Myers, 2017) or teacher noticing (Jacobs, Lamb, Philipp, 2010). It also fails to attend to the ways teachers might delay decision making during instruction for unseen reasons. That is, the choice to infer aims during instruction requires attention to the temporal or in-the-moment aims and fails to account for the possibilities of teachers delaying a decision for later in a lesson or the fact that teachers manage multiple goals simultaneously during instruction. Also, these goals can often be in conflict and are not always related specifically to teaching content. For example, teachers are always managing content goals, affective or social goals, justice or equity-based goals, and others. My attention to mathematics teaching from a specific set of core practices and grain sizes fails to take these goals into account. Further research could explore adding these goals to a hierarchical modular approach.

Regardless of these consideration, I have found hierarchical modularity to be useful for designing and researching teacher learning of practice. First, it addresses calls for a robust and shared conceptualization of practice to support design and research of teacher learning of core practices (Forzani, 2014; Jacobs & Spangler, 2017; Jansen, Grossman, & Westbroek, 2015; McDonald, Kazemi, & Kavanagh, 2013), and I have shown how this conceptualization might further the field in accumulating knowledge and building theory of teacher learning of practice. Second, given our use of the model with teachers in professional development and Jansen and colleagues (2015) promotion of a modular approach for prospective teacher learning, I suggest that modularity can provide a common conceptualization of practice that spans teacher preparation and professional development. Further research could explore the ways this model could be used in settings that bring together prospective teachers, with their limited conceptions of teaching, and teachers who hold existing conceptions of practice.

# CHAPTER III

# SECONDARY MATHEMATICS TEACHERS' RECOMPOSITIONS OF CORE PRACTICES OF AMBITIOUS TEACHING

Current mathematics education reform efforts highlight that students should learn meaningful mathematics in ways that support the development of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions toward mathematics (National Research Council, 2001). Reform efforts also encourage a form of teaching that leverages and builds upon students' mathematical thinking (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010; National Council of Teachers of Mathematics [NCTM], 2014, 2017, 2018). Often characterized as *ambitious teaching* (Jackson & Cobb, 2010; Kazemi, Franke, & Lampert, 2009; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; Thompson, Windschitl, &. Braaten, 2013), this vision involves teaching that is proactive and intentional in supporting students by problematizing ideas, eliciting and responding to the individual and collective mathematical thinking of students, and scaffolding classroom discussions toward teachers' learning goals (Kazemi et al., 2009; NCTM, 2014, 2017, 2018; Munter, Stein, & Smith, 2015; Smith & Stein, 2011). Mathematics learning opportunities that are constructed in these ways have positive implications for student learning (Boaler & Staples, 2008; Franke, Webb, Chan, Ing, & Battey, 2009; Stigler & Hiebert, 2004; Tarr, Reys, Reys, Chavez, Shih, & Osterlind, 2008). While promising, research suggests that ambitious teaching is often difficult to enact (Darling-Hammond & Synder, 2000; Lampert, 2010; Kennedy, 2005) and uncommon in mathematics classrooms (Stigler & Hiebert, 2004), resulting in teaching that is largely teacher-directed, focused on procedures, and marked with few opportunities for students to engage conceptually with mathematics (Munter, Stein, & Smith, 2015; Wiess, Pasley, Smith, Banilower, & Heck, 2003).

Recent efforts have made advances in identifying and describing content specific core practices of ambitious mathematics teaching (Core Practice Consortium, 2018; McDonald, Kazemi, & Kavanagh, 2013; NCTM, 2014; Teaching Works, 2016), and researchers are exploring ways to support teachers in learning to enact these practices (Boerst, Sleep, Ball, & Bass, 2011; Lampert et al., 2010, 2013; Webb, Wilson, Martin, & Duggan, 2015). Evidence suggests that, for teachers, relating ambitious forms of teaching to their existing practice can influence their instructional strategies (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Goldsmith, Doerr, & Lewis, 2014) and lead to changes in practice (Desimone, 2002). However, researchers have more recently argued that teachers' opportunities to learn ambitious teaching often focus on decomposing teaching into practices with little attention to the complementary work of recomposing practices back together to support students' learning (Jansen, Grossman, & Westbroek, 2015). In addition, a recent review of research on teachers' professional learning emphasized that the field lacks empirical evidence of the ways in which mathematics teachers work to incorporate core practices purposed for ambitious teaching into their existing frameworks for teaching (Goldsmith, et al., 2014).

In this mixed-methods study, I bring together a conceptual model of teaching using hierarchical modularity (Simon, 1965, 1973, 1996) and a situated theory of learning (Wenger, 1998) to investigate the ways teachers recompose three core practices to be more responsive to, and supportive of student learning. To do so, I retrospectively analyze four teachers' classroom lessons across their participation in two years of a practice-based professional development. I focus on the core practices of launching, monitoring, and discussing as a set of successive, large grain-size core practices central to teaching a task-based lesson, and two smaller grain-size core practices, pressing and orienting, as responsive moves essential to support students' collective learning of mathematics.

In what follows, I briefly review the literature on ambitious mathematics teaching and core practices to build a case for this study. I then outline a theoretical perspective on teacher learning in professional development using the lens of boundary encounters (Wenger, 1998) and a conceptual model for ambitious teaching using hierarchical modularity (Simon 1973, 1996). Next, I describe the professional development organized around cycles of investigating practice, explain the research method I used, and present findings related to changes in teachers' classroom enactments of individual core practices and the ways in which teachers recomposed each core practice over time to include pressing and orienting moves. I conclude with a discussion about the ways small changes in teachers' enactments of core practices and instructional moves can have profound effects on classroom instruction and how my conceptualization of teaching can support future research efforts focused on teacher learning of core practices.

## Core Practices of Ambitious Mathematics Teaching

Instruction seeking to meet the reform goals mentioned above has been characterized as ambitious (Forzani, 2014; Lampert et al., 2010), high quality (Munter, 2014), complex (Boaler & Staples, 2008), adaptive (Daro, Mosher, & Corcoran, 2011), and responsive (Jacobs & Empson, 2016), among other descriptors. While these characterizations may differ in some respects, they each attend to relationships between teachers, students, and content that are revealed as teacher's support students in instruction. In this study, I adopt the term *ambitious mathematics teaching* to describe instruction toward these reformed goals as well as to align with recent efforts to identify core practices of ambitious teaching and design learning opportunities to support teacher learning of practice (Campbell & Elliott, 2015; Forzani, 2014; Grossman, 2018).

In their influential study of relational professions, Grossman, Compton, Igra, Ronfeldt, Emily, and Williamson, (2009) detailed that decomposing complex professions into core practices can play a central role in clarifying the practices of a profession and supporting learners in understanding and enacting them. The idea of decomposing ambitious mathematics teaching has been taken up by researchers and teacher educators in recent years (Core Practice Consortium, 2018; Jacobs & Spangler, 2017; Lampert et al., 2013; NCTM, 2014; TeachingWorks, 2016), and Grossman and colleagues (2009) provide several criteria for identifying core practices central to ambitious mathematics teaching. They outline that core practices should be those that occur frequently in the work of teaching, are research-based and have impacts on student learning, attend to both teaching and students, maintain the complexity of teaching, and are learnable practices for both prospective and experienced teachers.

While supporting teachers in learning core practices has gained traction in teacher preparation, determining what constitutes a core practice and managing the appropriate grain sizes for learning them can be complex (Jacobs & Spangler, 2017). First, core practices can focus on interactive or non-interactive aspects of teaching. For example, designing a lesson is an important practice of mathematics teaching, but it is done prior to engaging with students, while launching a mathematics task takes place in direct relationship with students during instruction. Second, core practices can focus on content-specific aspects of teaching or broader aspects of teaching. For example, attending to issues of equity or implementing and maintaining norms for participation can be seen as interactive practices that may not be related to teaching a specific subject, while eliciting and responding to a students' mathematical idea would be a content specific practice. Third, core practices can vary in grain size. For example, launching a mathematics task or leading a discussion can be seen as a practice of larger grain size, while pressing a student to justify their reasoning or orienting students to one another's thinking can be seen as a practice of smaller grain size.

As Jacobs and Spangler (2017) note in their summary of research on core practices, the goal of this work "is not a consensus on practices but rather that the idea of core practices could become the field's vehicle for improvement" (p. 13). Thus, researchers and teacher educators seeking to use a core practice approach to support teacher learning must attend to which practices they choose to focus on, the grain sizes of these practices, and the relationships between practices to meet their goals for teacher learning. However, the field currently lacks a conceptualization of teaching that attends to the complexity of multiple practices of varied grain sizes and provides a way to understand the ways in which teachers bring practices together in their teaching. In this study, I focus on interactive, content-specific core practices and offer a way to conceptualize teacher learning and manage this complexity by investigating the ways in which teachers recompose individual core practices purposed for ambitious teaching.

## Theoretical Perspective

In this section, I bring together a theory of teacher learning and a conceptualization of practice to develop a framework to capture the complexity of teachers' enactments of core practices of ambitious mathematics teaching. I contend that when used together, these frames provide a means of characterizing the various ways teachers bring together their learning of practice in professional development with their existing system of teaching over time. For each frame, I provide an overview of its theory, outline its key components, and describe its uses. First, I introduce boundary encounters (Wenger, 1998) as a theory for understanding teacher learning across settings. Second, I outline hierarchical modularity (Simon 1965, 1973) to conceptualize ambitious mathematics teaching as a complex system. Finally, I bring these perspectives together to propose a frame for understanding how changes in different aspects of teachers' practice can be observed both within individual practices and across different grain sizes.

**Teacher Learning in Professional Development and Enactments**

Wenger (1998) introduced a theory of learning to address the social nature of our lived experiences in the world. From his perspective, knowledge is competence in a valued enterprise and knowing is “active participation in the practices of social communities and constructing identities” (p. 4) in relation to the enterprise. Important to this theory is the idea of *boundary encounter* to describe the ways communities come together to learn from one another. In a boundary encounter, members of different communities come together and use their respective practices to negotiate meaning. In doing so, each community introduces elements of their practice to the other community. In prolonged boundary encounters, new practices that are shared can emerge and precipitate the formation of a boundary community. As a way to collectively negotiate meanings together, these boundary practices inherit some elements of practice from each original community. Wenger (1998) outlines two processes central to the negotiation of meaning: participation and reification. Through participation, members of each community recognize mutuality in one another through “doing, talking, thinking, feeling, and belonging” (p. 56) as they interact together. Through reification, members project the meanings they are making to others through abstractions, tools, terms, or concepts in ways that bring “thingness” to practice and provide a focus for the negotiation of meaning. Together, participation and reification form a duality, and are thus, both distinct and complementary. Participation is required to negotiate meaning in community and reification is needed to give form to meaning in participation.

Three dimensions of practice that bring coherence as members negotiate meaning through participation and reification are mutual engagement, joint enterprise, and shared repertoire (Wenger, 1998). As participants mutually engage in collective activity they negotiate both collective and individual meaning. For this engagement to be more clearly defined as it relates to practice, participants engage in a communally negotiated focus – a joint enterprise. As members mutually engage in this joint enterprise over time, they create a shared repertoire of resources to support their ongoing negotiation of meaning. These resources include commonly understood and negotiated tools, routines, language, or actions.

As Wenger states, “learning is the engine of practice, and practice is the history of that learning” (p. 94). Thus, learning practice involves all three of the dimensions, as practice is both the context for learning and the goal. Evidence of learning can be observed as members negotiate meaning through their participation and reification in a boundary encounter, and also in the ways members incorporate elements of boundary practices into their own practice. Consequently, each act of participation and reification, whether in the boundary community or participants’ home communities, reflects aspects of both individual and collective learning.

Wenger (1998) describes collective learning of the boundary community as the development of shared meaning of practice and personal learning as identity development – the ways individuals navigate between the goals of their communities and their own personal goals. To more clearly operationalize individual learning, he presents three “modes of belonging”: engagement, imagination, and alignment. *Engagement* refers to the sustained involvement in the negotiation of meaning through participation in the boundary community. *Imagination* refers to the images and connections individuals make to relate their own experiences to those of the boundary community, and the ways they imagine new possibilities or alternatives for themselves. The work of imagination requires that one take risks, explore alternative practices, and try on new identities. Imagination is fostered through activities that promote connections between the practices of individuals and the boundary community, and opportunities to explore, rearrange, repurpose, or add new aspects to ones existing practice. Thus, imagination not only "support(s) the process of acquiring knowledge, but also offer(s) a place where new ways of knowing can be realized" (p. 215). *Alignment* represents the ways in which individuals coordinate their practice to align with the those of the boundary community and their own goals for themselves.

Combining engagement, imagination, and alignment in different ways brings into focus different opportunities to learn (Wenger, 1998). Combining engagement and alignment, individuals bring their own perspectives to the boundary community, coordinate them with respect to the shared aim of the boundary community, and over time may develop understandings, goals, and practices that align to the boundary community. Combining engagement and imagination, individuals identify with the goals of the boundary community through engaging in them with others and imagine other possibilities for their own practice as they consider their existing practices and new possibilities for the future. Combining imagination and alignment, individuals reconcile their ideas for new possibilities for practice with the realities of their existing practice as they navigate between imaginative and enacted practice.

The strength of this theory is that it blurs dichotomies between talking and doing, ideals and reality, and the boundary between communal and personal as individuals make meaning of practice through engagement, imagination, and alignment (Wenger, 1998). However, this theory does not address the complexities of professional practices nor offer a way to manage multiple, interconnected practices. I now introduce a conceptual model for teaching that uses hierarchical modularity to address these shortcomings and then bring the two theories together to frame the study.

**Hierarchical Modularity**

Mathematics teaching is a complex practice. In the last decade, researchers and teacher educators have endeavored to parse teaching into core practices to support teachers in learning to enact ambitious teaching. While emerging research on a core practices approach is promising, Jansen, Grossman, and Westbroek (2015) argue that professional learning opportunities that focus on decomposing teaching into core practices often fail to attend to the complementary work of recomposing the enactment of these practices back together to support student learning. Moreover, they also contend a core practice approach has focused heavily on the skills of teaching, with less consideration of the ways teachers aims for enacting these skills develop. They proposed that the field explore the use of hierarchical modularity (Simon, 1973) as a potential way to resolve these tensions.

Simon (1965, 1973) offered hierarchical modularity as a way to conceptualize complex systems for learning and research. He claimed that most systems can be classified as hierarchical, are “nearly decomposable”, and can be conceptualized as a collection of “localized subsystems,” with properties that are both specific to each subsystem and relate to other subsystems within the larger system. Although mainly used in research on physical systems, Simon asserted that hierarchical modularity is useful in modeling social phenomena, and it has been used in design and research of complex systems in organizational management (Sanchez & Mahoney, 1996), biology (Kashtan & Alon, 2005), engineering design (Sosa, Eppinger, & Rowles, 2007), clinical psychology (Chorpita, Daleiden, & Weisz, 2005), and cultural change (Wimsatt, 2013), among other fields.

Simon (1973) describes hierarchical modular systems from both a state and process perspective to highlight the identifiable characteristics of the system and the actions embedded within a system. Extending and clarifying this work, Bethel and Richardson (2010) characterized these systems from both a structural and functional perspective. Structurally, hierarchical modularity is simply a way to describe a system, identify subsystems within the system, and mark connections between and across these subsystems. Functionally, hierarchical modularity describes the effects that subsystems have on different parts of the system and the aims or “emergent properties” that can be inferred when these subsystems are made visible.

Three characteristics are central to understanding hierarchical modularity and its use in researching complex systems (Simon, 1965, 1973). First, connections exist within a subsystem that do not necessarily exist across subsystems or on different levels of the system, a feature that Jansen and colleagues (2015) refer to as “internal coupling.” The second characteristic, “horizontal coupling,” describes the connections that exist across subsystems. The third characteristic, “vertical coupling” signifies that within a system, subsystems at lower levels can occur in multiple higher-level subsystems.

Scholars reason that attention to both the structural and functional perspectives of complex systems plays several important roles. First, it maintains a focus on the duality when managing and adapting within a system. It also supports an analysis of evolution and change within a system and allows for a better understanding of the ways small changes within different subsystems propagate to produce large changes within the system (Baldwin & Clark, 2000; Holland, 2012; Jansen, Westbroek, Doyle, & van Driel, 2013; Simon, 1996). While useful for design and framing questions for research, when examining complex social systems, a modular approach fails to attend to the meanings communities and individual are making of the practices that underlie the complex system. Wenger’s (1998) notion of boundary provides a way of understanding learning of practice, however, it does not attend to the challenges of managing complex systems of practice.

## Conceptualizing Ambitious Mathematics Teaching for Practicing Teachers

I now bring these frameworks together for a study of teacher learning of core practices of ambitious teaching. I first use hierarchical modularity to describe the set of core practices and instructional moves used in this study. I highlight the ways using both a structural and functional perspective can support an analysis of teacher learning of core practices. Next, I bring this model together with imagination and alignment to provide a theoretical explanation of the ways individual teachers enact core practices of ambitious teaching over time.

From a structural perspective, I conceptualize teaching as a complex system, large grain-size core practices as a set of subsystems, and instructional moves as a set of smaller grain-size core practices made visible during instruction. A benefit of this approach to conceptualizing practice for teacher learning in professional development is that all teachers come to professional development with an existing system of teaching, including practices they enact and instructional moves they employ. Thus, this model attends to and respects teachers' current knowledge and practice while also identifying various aspects of their practice to consider as they work to improve in their teaching and support student learning.

For this study, I focused on three large grain-size core practices that have been both sites for research and shown to support student learning: *launching* a mathematics task (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013), *monitoring* students engagement in the task (Stein, Engle, Smith, & Hughes, 2008), and *discussing* the task with the whole class toward a mathematical goal (Chapin, O'Connor, & Anderson, 2009). In the broader design of our professional development, we addressed five smaller grain size core practices: *explaining* a mathematical idea, *revoicing* students' contributions, *probing* to uncover students' thinking, *pressing* students to justify their reasoning or consider alternative mathematical ideas, and *orienting* students to one another's mathematical thinking. Some teacher educators have noted a tension that results from not having a shared conceptual model of teaching with accompanying technical language to describe it . In the professional literature, some refer to the smaller actions teachers' employ with a specific goal as an instructional (Harris, Phillips, & Penuel, 2012) or teaching (Jacobs & Empson, 2016) move, while others call these actions moves and also refer to them as core practices (Grossman, 2018). I chose to refer to the smaller core practices as *instructional moves* made visible during instruction. This language choice distinguished levels of the subsystem and provided a way to discuss these smaller practices as nested and connected within and across the larger practices.

In this study, I focus on pressing moves (Kazemi & Stipek, 2001) and orienting moves (Boaler & Brodie, 2004; Boaler & Staples, 2008; McDonald, Kazemi, & Kavanagh, 2013) as these moves have been shown to support student learning and are dependent on responding to prior interactions with students' mathematical thinking. When using an instructional move, one must attend to both the object of the move and its goal. For example, to use an explaining move, the object of that move might be a students' written mathematics, and a goal for explaining might be to ensure the student understands a procedure. Using this move does not necessarily require prior knowledge of students' thinking, as a teacher could look at a students' written mathematics, recognize an aspect of their mathematics to address, and explain a procedure to the student without ever inquiring about their thinking. Conversely, to use a pressing or orienting move requires that one build from prior knowledge of students' thinking. For example, orienting a student to another students' mathematical thinking requires both an understanding of each students' thinking and a goal for orienting them to one another, such as the desire to have the students relate their ideas together. I conjectured that recomposing enactments of each of the larger practices to include pressing and orienting moves would support teachers in progressing students' mathematical thinking across a lesson toward their goal.

From a functional perspective, I conceptualized the aim for enacting a practice or move as emergent properties that can be inferred when subsystems are made visible during instruction. To differentiate between aims at different levels of the system, I defined the reason for enacting a large-grain size core practice as its *purpose* and the reason for enacting an instructional move as its *goal*. As shown in Figure 3, I organized the practices and moves of focus in this study and their respective aims to conceptualize ambitious mathematics teaching as a hierarchical modular system.

| **System** | **Ambitious Mathematics Teaching** | | | *Vision* for mathematics teaching |
|---|---|---|---|---|
| **Core Practices** | **Launching** | **Monitoring** | **Discussing** | *Purpose* for each practice |
| **Instructional Moves** | **Probing Revoicing Explaining**<br>***Pressing Orienting*** | | | *Goal*(s) for moves |

Figure 3. Hierarchical Modularity of Ambitious Mathematics Teaching

Though there are dependent connections across core practices (i.e., the outcomes of launching a task may affect the whole class discussion), the characteristics of internal coupling and vertical coupling allows one to temporarily suspend attention to these relationships in order to focus on individual core practices and their purpose (Jansen et al., 2015; Simon, 1973). For example, teachers and researchers can examine the core practice of monitoring without attending to what might have occurred prior to monitoring during launching, or what might happen later in the lesson. Vertical coupling permits a focus on the instructional moves teachers use when enacting a single core practice while ignoring how the same move is used within a different core practice. Instructional moves can therefore be used across multiple core practices, yet the goals for these moves may differ across practices. For example, teachers and researchers could examine goals for using pressing moves during a discussion, while recognizing that the goals for this move may differ during other aspects of a lesson, such as the goals for using pressing moves during the launch of the task.

In summary, conceptualizing teaching using hierarchical modularity provides a way to investigate teacher learning by investigating core practices and purposes, or instructional moves with goals as different but dependent units of teaching. Evidence of learning could be the addition of new core practices, the use of an existing core practice with a more ambitious purpose, adopting and using new moves, or having a more ambitious goal for moves already a part of one’s teaching. First, researchers could investigate the ways teachers bring together sets of large-grain size core practices toward a broader vision of ambitious teaching. Second, researchers could investigate the ways teacher recompose their enactments of individual core practices toward a purpose that is more aligned with ambitious teaching. Third, researchers could investigate the ways teachers use new moves or modify their goals for existing moves to recompose an individual core practice in ways that are more supportive of student learning.

I now return to my theoretical perspective of individual learning (Wenger, 1998), focusing on combinations of engagement, imagination, and alignment. Wenger states that each of these modes of belonging are "important ingredients" of learning but that each mode has individual shortcomings; thus combining them "create[s] richer context for learning" (p. 216) in professional development. As teachers participate in professional development, they engage in potentially new goals and purposes and align their current practices and moves as they make public their evolving conceptions – coordinating them with respect to their own practices, the goals of the professional development, and the boundary communities' shared conceptions of the practices. As teachers approximate enacting core practices in professional development (e.g. rehearsals), they engage in the practices of the boundary community and imagine new possibilities for themselves that may be more ambitious. In teachers' classroom lessons, the focus of this study, they align their imaginative practice to their existing practice as they try out new possibilities for practices and reconcile them with their existing practices.

## Methods

This mixed-methods study is part of a larger design experiment examining secondary mathematics teachers' learning of ambitious teaching in professional development. It focuses on two implementations of a yearlong professional development program designed to share research on mathematics instruction with teachers, provide teachers with opportunities to consider their existing practice, approximate practices of teaching in ways that may be more ambitious and supportive of student learning, and try out new possibilities for practice. Broad measures of instructional quality conducted by the project’s evaluation team showed that all teachers improved in their ability to teach task-based lessons (Duggan & Jacobs, 2017). In this study, I retrospectively examine four teachers’ enactments of individual core practices of ambitious teaching in five lessons spread over two years. I aim to better understand the ways teachers enacted a set of three core practices and recomposed these practices to use responsive instructional moves by answering the questions, *In what ways, and to what extent, did teachers recompose the core practices of launching, monitoring, and discussing, over the course of two implementations of professional development?*

**The Professional Development Context**

Though a full presentation of the professional development is beyond the scope of this paper, I provide an overview and focus on several key ideas that are necessary for understanding my examples and findings. Our design assumed that teachers come to professional development with expertise about teaching and students and have an existing complex system of practice to work from. Throughout both implementations (2015-2016 and 2016-2017), members of our research team and participating teachers engaged with a conception of teaching that allowed us to highlight core practices of ambitious teaching while also respecting teachers’ existing practices. We created opportunities for teachers to build from and challenge their existing system of teaching, including the practices and instructional moves they typically drew upon in instruction and their aims for doing so. In doing so, teachers explored opportunities to repurpose aspects of their existing practice for more ambitious purposes and try out new practices and moves in rehearsals. As facilitators, we established and maintained norms for participation that enabled teachers to negotiate meaning of core practices and instructional moves that more closely attended to students' mathematical work and thinking. As researchers, we attended to how teachers learned the core practices and instructional moves to both understand teacher learning of ambitious teaching and refine the professional development design over subsequent iterations.

Prior to the first implementation of the professional development, we met three times with teachers who had shown interest in participating to broadly share with them representations of ambitious mathematics teaching. In each of these three meetings, teachers experienced an ambitious mathematics lesson as we facilitated their engagement with cognitively demanding tasks. In doing so, we provided teachers with a beginning representation of what ambitious mathematics teaching could look like as a teacher, and as a learner. After these three meetings, teachers were asked to teach a lesson using a task they experienced from the first meeting to capture a baseline of their current practice. The task they were asked to teach, the Zipline Task (Appendix B), was designed for students to explore the relationship between the length of a wire stretched between two fixed towers and connected to the ground in between the towers at one movable point, creating two right triangles. The goal of the task was to minimize the sum of the lengths of the two hypotenuses created by the wire by determining the appropriate location for the movable point. This task was designed to be approachable for all students and could be solved by creating a scale drawing, building a table, using transformational geometry, using trigonometric ratios and angle measures, or creating and finding the minimum values of a function.

Each implementation was built from a consensus view for effective professional development (Darling-Hammond et al., 2009; Elmore, 2002) and designed for a 12-month period consisting of a 60-hour summer institute followed by 20 hours of follow-up meetings throughout the school year. As designers, our team was guided by the assumption that an approach grounded in representing, decomposing, and approximating (Grossman et al., 2009) ambitious mathematics teaching could support practicing teachers in learning to enact ambitious teaching.

For the summer institutes, we developed sequences of professional learning tasks that focused on representing the work of ambitious teaching and decomposing teaching to make both the core practices and instructional moves public for analysis and discussion. For the practices of launching, monitoring, and discussing, we formalized the discussion of these practices with frameworks we developed that highlighted the purpose of the practice and corresponding goals for moves that teachers could use to support their enactment toward the purpose (Appendix A). Using the frameworks, teachers approximated practices of ambitious teaching with artifacts of practice (e.g., analyzing student work or classroom videos). The practices and moves addressed in the professional development are outlined in Figure 4.

<table>
<tr><th>Core Practices of Focus in the Professional Development</th><th colspan="2">Instructional Moves</th></tr>
<tr><td>Selecting Tasks & Establishing Learning Goals</td><td colspan="2" rowspan="3"></td></tr>
<tr><td>Classifying & Adapting Tasks for Ambitious Teaching</td></tr>
<tr><td>Anticipate Students’ Mathematical Approaches</td></tr>
<tr><td>Launching Cognitively Demanding Tasks</td><td rowspan="3">Pressing<br>Orienting</td><td rowspan="3">Revoicing<br>Probing<br>Explaining</td></tr>
<tr><td>Monitoring Students Engagement with Tasks</td></tr>
<tr><td>Facilitating Whole Class Mathematics Discussions</td></tr>
</table>

Figure 4. Overview of Core Practices and Instructional Moves in the PD

A key feature of the professional development was the use of rehearsals for the core practices that occur during instruction (italicized in Figure 4). After approximating the practices of launching, monitoring, and discussing in the summer institutes, each teacher rehearsed each of the practices. During all rehearsals, a researcher served as a facilitator who worked to make teachers’ instructional decisions public by stopping the rehearsal at various times to elicit the retrospective decisions they made or their conjectures about future actions they could take in the rehearsal. A detailed description of the rehearsal design is reported elsewhere (Webb, in preparation a).

Throughout the school year, teachers met bi-monthly with the research team after school to relate their learning from the summer institute to their teaching during the school year. Through tasks such as analyzing a recording of their own teaching, collaborative planning for instruction, or rehearsing core practices of upcoming lessons, these activities during the school year were designed so that teachers could set their own goals related to their teaching practice and have support to achieve them.

## Participants

The research team partnered with one midsized, rural school district in the southeastern United States. The district served approximately 13,000 racially, ethnically, linguistically, and socioeconomically diverse students; the student population was 62% Caucasian, 20% African American, 11% Hispanic, 1% Asian, and 5% multi-racial, and 63% of students in the district qualified for free or reduced lunch. At the time of the first implementation of the profession development, 57% of students were rated as proficient in the first high school mathematics course—which in this case was an integrated course containing units focused on the domains of algebra, function, geometry, and statistics.

Across the two implementations, 19 mathematics teachers who taught high school mathematics courses volunteered to participate, receiving a stipend for their participation. Eight of these teachers participated in both implementations. Of these eight teachers, one taught high school courses to middle school students, one taught in a specialized magnet school, and two did not participate in all of the professional development activities. The remaining four teachers, Dawn, Carla, Sara, and Brenda, completed the task-based lesson prior to the first implementation, participated in all activities, taught high school mathematics in a traditional public school, and are the case teachers chosen for this study.

## Research Design

This study used a two-phase, concurrent mixed-methods design (Creswell & Plano Clark, 2011) to investigate teachers' enactments of three successive core practices of ambitious teaching over the course of two implementations of professional development. My examination consisted of two phases. In phase one, I used my framework to concurrently analyze quantitative and qualitative data on teachers' enactments of each core practices. To do so, I examined transcripts of each teacher's lessons with a focus on the practices of launching, monitoring, and discussing, the instructional moves the teacher used, and their purposes and goals for enacting the core practice and moves.

In phase two, I used results from phase one to look across each individual practice and qualitatively analyze changes in each teacher's enactments of launching, monitoring, and discussing, including their purpose for enacting the core practice, their use of pressing and orienting moves, and their goal for these moves to characterize the ways teachers worked to recompose each core practice over time. The first phase was essential in characterizing teachers' enactments of each of the core practices and their use of instructional moves for each lesson. The second phase was essential in characterizing the ways their enactments of each core practice were recomposed toward a more ambitious purpose across their five lessons. The combined results of the two phases assisted me in understanding the ways changes at lower subsystems of practice related to changes in higher subsystems across teachers' enactments of core practices of ambitious teaching.

**Data Sources and Analysis**

Data for this study included artifacts from each of the teachers' lessons across the two implementations of the professional development. Each teacher in this study taught the Zipline Task in Spring 2015, prior to participation in the professional development. After each summer institute, teachers taught two lessons, one using the Zipline Task in the fall semesters (Fall 2015, Fall 2016) and a cognitively demanding task of their choice in the spring semesters (Spring 2016, Spring 2017), for a total of five lessons per teacher. In total, data included videos and transcripts of 20 lessons.

**Phase one.** For phase one, conceptualizing ambitious mathematics teaching as a hierarchical modular system (Figure 3), I analyzed teachers' enactments of the three core practices as well as the instructional moves they used. For the larger core practices of launching, monitoring, and discussing, I used the characteristics of internal coupling to focus individually on these practices for analysis. For the instructional moves of pressing and orienting, I used the characteristics of vertical coupling to investigate the ways teachers used these moves within each of the larger practices.

As part of our larger research, the project team quantitatively analyzed teachers' whole lessons using a subset of the Instructional Quality Assessment rubrics (IQA) (Junker, et al., 2005), an existing validated measure of instructional quality. The Academically Relevant Questions (ARQ) rubric was used to assess the overall rigor of teachers' questions and the degree to which the teacher provided opportunities for students to elaborate and explain their work, thinking, or mathematical ideas, on a scale from 0-4. To use this measure to analyze instruction as a hierarchical modular system, I modified this rubric to broadly measure the degree to which teacher met the purpose of the practice promoted in the professional development (Appendix C), thus obtaining an ARQ score for each core practice.

Next, to analyze each core practice from a vertical coupling perspective, I modified two additional IQA rubrics (Junker, et al., 2005) used by the larger project team to assess teachers' pressing and orienting moves within each core practice. The Teachers'

Press (TP) rubric was designed to assess the degree to which teachers pressed students for conceptual explanations or to extend their mathematical thinking during a whole class discussion, on a scale from 0-4. The Teacher's Orienting (TO) rubric was designed to assess the degree to which teachers connected students' contributions and showed how these contributions related to each other, on a scale from 0-4. To use these measures to analyze instruction as a hierarchical modular system, I modified each rubric so that they could be used to assess the quality of teachers' pressing and orienting moves within each core practice (Appendix C), obtaining a TP and TO score for each core practice.

While capturing the quality of teachers' enactments of each core practice and instructional moves was important, the rubrics did not capture whether quality was due to a large number of pressing or orienting moves or a more limited number of moves of higher quality. To better understand the moves teachers chose to use, a quantitative count of the moves highlighted in the professional development was obtained for each core practice within each lesson. To do so, I specified a teachers' talk turn as the unit of analysis and coded for each move using a codebook developed by the research team that drew upon research on teachers' instructional moves (Appendix D).

Four independent coders were trained to score lessons using the IQA rubrics and identify instructional moves. Members of the research team served as coders and achieved 88% interrater reliability on the IQA rubrics and 84% interrater reliability on instructional moves. As one of the coders, upon reaching this level of agreement, I scored each core practice and coded for instructional moves for the data set of the four teachers of focus for this study.

Finally, I analyzed the purpose of teachers' enactments of each of the core practices and the goals for their use of pressing and orienting moves by drawing upon the functional perspective of hierarchical modularity and emergent properties, as aims that can be inferred when subsystems are made visible during instruction. I used constant comparative methods (Glaser, 1965; Strauss & Corbin, 1998) to analyze transcripts of teachers' enactments of the core practices that were coded for instructional moves to infer the teachers' purpose for the practice and their goals for using pressing and orienting moves. When needed, I also analyzed video data to clarify inferences, better understand the context of interactions between teacher and students, and attend to any non-verbal cues that could support or clarify inferences. I first focused broadly on the enactment of each core practice to infer its purpose, and then looked across teachers' pressing and orienting moves within each practice to infer a "representative" goal a teacher had for enacting pressing or orienting moves.

***Inferring the purpose of a practice.*** Inferring the purpose for a teachers' enactment of a core practice involved a constant comparative approach of weighing the purpose for the practice promoted in the professional development described by the frameworks (Appendix A) against the ways the practice was enacted. For example, the proposed purpose for monitoring was to support all students in productively engaging with an instructional task in ways that advance the learning goal of the lesson. To do so, the framework outlined a set of goals for moves to support the meeting of this purpose, such as encouraging students to think more deeply about the mathematics. To illustrate my process for inferring the purpose of a practice, I use an example of one teacher's enactment of monitoring in her lesson in Spring 2015, before attending the professional development. The teacher walked around the classroom and spoke with small groups, trying to understand their mathematical thinking. When students shared mathematical conceptions that hindered their ability to progress with the task, the teacher left the group to think on their own and did not respond to their requests for support. In this case, I inferred the purpose for monitoring for this teacher at this point in time was to provide students with the opportunity to refine their thinking independently or with their peers.

***Inferring the goal for moves.*** I inferred and summarized a "representative" goal a teacher had for enacting pressing or orienting moves within each practice for two reasons. First, because this was a retrospective study, I could not ask teachers questions about the goal for their moves, and thus was limited to what I could infer from the transcripts or reconcile with their classroom videos. Relatedly, teachers simultaneously manage multiple goals during instruction (e.g. affective, social, and content goals) and I was unable to attend to these possibilities. Thus, I was more interested in an overall characterization of these moves rather than a moment-by-moment analysis of the multiple micro-level goals teachers may have for a move that are often metacognitive and non-visible.

Inferring a representative goal for pressing and orienting moves similarly involved a constant comparative approach of weighing the definition of the move described in the codebook (Appendix D) against the coded moves. To infer a representative goal for pressing and orienting moves, I looked across the moves used within each practice to summarize a goal for teachers' typical aims for using the moves within each practice. This comparison further specified the ways teachers' goals for moves might relate to their enactment of the practice and its corresponding purpose.

For instance, the definition of pressing in the codebook was purposefully broad to capture several ways in which teachers could press students. As one example, a teacher leading a discussion used seven pressing moves that were generally of the form, "Can someone make this like we are used to seeing in function notation?" and "Can we write this (linear function) differently but so that it means the same thing?" These were coded as pressing moves because the teacher asked students to extend their thinking about a mathematical idea, in this case to relate their existing work to a traditional algebraic representation of a linear function. From these moves, I described the representative goal for this teachers' pressing moves when leading a discussion as pressing to focus on superficial features of mathematical representations. This more specified goal allowed me to compare to it other representative goals such as pressing a students' emerging strategy to support them in generalizing the mathematics. Pressing and orienting moves represented only 11% of the total number of moves used across all teachers' lessons and for the most part accounted for less than ten moves within each practice during a lesson. Thus, inferring representative moves within a practice was manageable. In instances where teachers used a larger number of moves within a practice, it was most often during monitoring as they engaged with small groups and they typically used similar moves within their interactions with each group, thus facilitating my inferences.

To organize these data in a way that supported my two-phase approach, I created lesson summaries for each lesson that included the coded data and results from phase one analysis (see Appendix E for example). These lesson summaries broke the whole lesson up into three sections, each representing one of the core practices (i.e., launching, monitoring, discussing). Within each of these sections, I included the IQA rubric scores, the quantitative counts for each move, the inferred purpose for enacting the core practice, and the inferred goal for using pressing and orienting moves.

**Phase two.** Using these lesson summaries, I used a constant comparative approach (Glaser & Strauss, 1967; Strauss & Corbin, 1998) to look at each teachers' five enactments of each core practice, individually focusing on one teacher before moving on to the next. First, I looked across the lesson summaries to characterize any changes in a teachers' enactment of each practice, attending to whether they added new practices to their lesson or enacted an existing practice with a more ambitious purpose. Next, I looked across the lesson summaries to characterize any changes in teachers' use of pressing or orienting moves, attending to whether there were additions of new moves or the use of an existing move toward a more ambitious goal. Finally, I looked collectively at these characterizations to summarize the ways in which teachers brought instructional moves together within each practice over time to recompose the practice in ways that were more supportive of student learning by conceptually drawing upon both internal and vertical coupling from my framework.

## Findings

Results from this mixed-methods study indicate that, over time, each of the four teachers recomposed the practices of launching, monitoring, and discussing across their five lessons; however, their purpose for enacting the practices, the extent to which they recomposed their enactments to use pressing and orienting moves, and their goals for these moves differed. Recall that across the two implementations of the professional development, evaluation studies demonstrated from broad measures of instructional quality, that all teachers grew in their ability to teach task-based lessons (Duggan & Jacobs, 2017). In addition, elsewhere I have reported on the ways teachers can recompose the practices of launching, monitoring, and discussing to teach more ambitiously, conceptually drawing upon horizontal coupling (Webb, in preparation a). In this study, I was interested in how teachers' enactments of individual practices progressed across over the course of the professional development, and the ways they recomposed these practices over time to include pressing and orienting moves. To organize these findings, I present them by each core practice, beginning with launching, then monitoring, and finally, discussing to investigate teacher learning with each practice and demonstrate how my conceptual model can attend to the complexities of practice vertically across levels of practice for multiple practices.

For each practice, I begin by sharing the quantitative data. I share teachers' ARQ scores to present the degree to which teachers met the purpose of the practice promoted in the professional development. Next, I share each teacher's TP scores assessing the degree to which they pressed students for conceptual explanations or to extend their thinking, and the percentage and number of moves they used when enacting the practice. Then, I share each teachers' TO scores assessing the degree to which teachers connected students' contributions and showed how these contributions related to each other, and the percentage and number of moves they used when enacting the practice.

To support an understanding of the findings, I present the quantitative data of teachers' pressing and orienting moves as both a percentage of total moves used within that practice as well as the number of moves used for several reasons. First, because the amount of time teachers chose to take when enacting each practice varied (i.e., a teacher might take 5 minutes to launch a task while another might take 8 minutes), a single representation (i.e., number or percentage) does not adequately capture teachers moves. In some cases, teachers' use of pressing and orienting moves increased or decreased as a percentage of total moves used when enacting one core practice, but did the opposite when represented as a count of moves. In other cases, an increase in the percentage or number of moves a teacher used did not relate to the quality of these moves. Thus, sharing two representations of "quantity" of moves facilitates a better understanding of the ways teachers used pressing and orienting moves over their five lessons.

As context, I also coded for probing, explaining, and revoicing moves. Briefly, across all four teachers and their five lessons, the average percentage of these moves used were *probing* (53%), *explaining* (21%), *revoicing* (15%), representing on average 89% of the 5,300 coded teachers moves. For this analysis, I restricted my focus to the use of pressing and orienting moves based on my conjectures about teachers' use of responsive moves that are contingent of prior knowledge of students' thinking and the impacts these moves can have on instruction.

**Launching Mathematics Tasks**

Across all four teachers, their launches improved and each teachers' purpose for launching became more closely aligned with the purpose of launching promoted in the professional development as shown in Table 1, which was to ensure that students understood the mathematical goal of the task and were able to begin engaging in the mathematics of the task (Appendix A, C).

Table 1

Launching Tasks ARQ Scores – Meeting the Purpose of Launching

| **Academically Relevant Questions Rubric (0 – 4)** | | | | |
|---|---|---|---|---|
| **Enactment** | **Dawn** | **Carla** | **Sara** | **Brenda** |
| **1** | 2 | 4 | 0 | 0 |
| **2** | 3 | 4 | 4 | 1 |
| **3** | 3 | 3 | 3 | 4 |
| **4** | 3 | 4 | 4 | 3 |
| **5** | 4 | 4 | 4 | 3 |

While all teachers' enactments of launching improved, their use of pressing and orienting moves differed, as quantitively represented in Table 2 and Table 3 respectively. For Dawn, pressing and orienting moves were for the most part not evident until her fifth enactment. For Carla, she sparingly used pressing moves and often used orienting moves throughout her enactments. For Sara, she used pressing and orienting moves in both her fourth and fifth enactment, while Brenda rarely used either move in her enactments. In what follows, I use these quantitative descriptive data, and the qualitative lesson summaries to share findings of the ways each of the four teachers' enactments of the core practice of launching evolved over their five enactments, attending closely to their purpose for launching and their goals for using pressing and orienting moves.

Table 2

Launching Tasks – Pressing (TP) Scores

| Teacher Press / % Pressing Moves | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Enactment** | **Dawn** | | **Carla** | | **Sara** | | **Brenda** | |
| **1** | 0 | 0% (0) | 0 | 1% (1) | 0 | 0% (0) | 0 | 0% (0) |
| **2** | 0 | 0% (0) | 2 | 4% (1) | 0 | 0% (0) | 0 | 0% (0) |
| **3** | 0 | 0% (0) | 0 | 0% (0) | 0 | 0% (0) | 2 | 2% (1) |
| **4** | 0 | 0% (0) | 2 | 3% (2) | 2 | 8% (3) | 0 | 0% (0) |
| **5** | 3 | 7% (4) | 3 | 7% (2) | 2 | 9% (5) | 0 | 0% (0) |

Table 3

Launching Tasks – Orienting (TO) Scores

| Teacher Linking / % Orienting Moves (# of moves) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Enactment** | **Dawn** | | **Carla** | | **Sara** | | **Brenda** | |
| **1** | 0 | 14% (1) | 4 | 16% (15) | 0 | 0% (0) | 0 | 0% (0) |
| **2** | 0 | 0% (0) | 3 | 13% (3) | 0 | 0% (0) | 1 | 5% (2) |
| **3** | 0 | 11% (1) | 0 | 7% (1) | 0 | 0% (0) | 0 | 0% (0) |
| **4** | 0 | 0% (0) | 3 | 7% (5) | 2 | 13% (5) | 0 | 0% (0) |
| **5** | 2 | 5% (3) | 2 | 10% (3) | 2 | 3% (2) | 0 | 0% (0) |

**Launching mathematics tasks – Dawn.** For Dawn, the purpose of launching in her first lesson was to present the task to students and let them explore the context and goal of the task on their own. For example, when a student asked a question related to an important feature for students to understand to engage with the task, she stated, “That’s a question you will want to talk to your group about.” Across her lessons, her purpose for launching changed to focus on providing opportunities for students to share their emerging mathematical thinking about the task and supporting students’ understanding of the task’s context and goal. Across her remaining four lessons, Dawn elicited students’ understanding of the context and mathematical goal of the task with questions such as, “Are there any restrictions or guidelines?” (2nd enactment), “Does another group want to share key ideas?” (3rd enactment), “Is there at least one person in every group who knows enough to get started?” (3rd enactment), “Do you think you know how to get started?” (4th enactment), and “What is one thing your group noticed?” (5th enactment).

While Dawn’s purpose for launching became more aligned with the purpose promoted in the professional development (Table 1), she did not use pressing and orienting moves until her fifth lesson (Tables 2 & 3). In this lesson, students engaged in a growing pattern problem that could be represented as a linear relationship. During her launch, she had students develop conjectures in small groups about what they noticed across a sequence of blocks arranged in a growing pattern.

> Dawn: What did you write down, S1?
> S1: That it goes up one and out one each time.
> Dawn: Up one and out one. S2, what did you write down? (*S2 explains their thinking*)
> Dawn: Does he have something similar to what you have or different?...Can you share your ideas with your partners? (*orienting move*)
> *Dawn approaches another group and after they share their thinking.*
> Dawn: Can you find a different way of expressing the change?...Make a list and see how many you can come up with. (*pressing move*)

In this example, Dawn oriented students to each other’s ideas and pressed students to consider multiple approaches or strategies prior to having them work on solving the task.

**Launching mathematics tasks – Carla.** Across her five lessons, Carla maintained fairly consistent launches that ensured students understood the context and mathematical goal of the task and were able to begin with the mathematics of the task

(Table 1). The degree to which these launches met the purpose promoted in the professional development related to her use of orienting and pressing moves. In Carla's launch in her third lesson, which was scored a 3, she did not use pressing or orienting moves. In this launch, students worked to generalize a pattern of the "size of a dollar bill that has been shrunk by a reduction of 30% - 3 times, 6 times, 9 times, n times." When she brought the class together prior to having them begin to work on the task, she engaged the class in what can best be categorized as an initiate, respond, evaluate approach to progress the lesson forward, stating,

> Carla: Let's hear some of your ideas. S1, what is the problem asking us to do?
> S1: To find the um, how many different sizes for six, nine, and n.
> Carla: For six, nine, and n. So, we are going to find a couple different answers, and then we are going to look for a couple different things. S2, is there anything important in the problem that we need to know about? How are you going to change the size of it?
> S2: Reduce it.
> Carla: You're going to reduce it by…?
> S2: 30%
> Carla: By 30%. So, you are going to reduce it by 30% 3 times, 6 times, 9 times, and n is just a general... Alright, so I want you to go ahead and get started.

In this example, Carla elicited students' understanding of the mathematical goal and verified that they understood the need to reduce the figure by 30% each time. In her other enactments, which were all scored a 4, she predominantly used orienting moves, as shown by a sample of her moves from her first and fourth enactments shown in Figure 5. For these orienting moves, evidence suggests that Carla's goal for using these moves was to connect students' emerging mathematical ideas, approaches, or representations about the task to one another, and in doing so, she further supported students in beginning to collectively engage in the mathematics of the task.

| Lesson | Orienting Moves |
|---|---|
| 1 | How about Jen, does she have the same picture as you?<br>Talk to each other about why you chose Pythagorean Theorem<br>Talk as a group and decide what approach are you going to take to solve this problem.<br>Do you agree with her picture? |
| 4 | I want you two to talk for a minute because your drawings are different. What is similar, what is different?<br>Is there anything he has that is different than yours? |

Figure 5. Carla's Orienting Moves During Launching.

**Launching mathematics tasks – Sara.** Sara's purpose for launching in her first lesson was to get students working on the task as quickly as possible. She began by handing out copies of the task to students, read the problem aloud stating, "I am not going to answer any questions at this point," and had students begin to work on the task. Across the remainder of her lessons, her purpose for launching changed to include opportunities for students to elaborate their thinking and supports for students to understand both the context and goal of the task. In Sara's third launch for example, students considered the task in small groups. Afterwards, she brought the class together and had the following exchange:

Sara: S1, can you reiterate the three things I wanted you to do?
S1: Do you need anything to be clarified, do you understand the question, do you have a starting point.
Sara: Is there anything you need clarification about?
S2: Are there two ziplines both coming from the towers?
*Sara explains to class.*
Sara: Anything else, clarification?

*Sara answers a few other clarifications.*
Sara: Can you all restate the question, what is it we are trying to find?
S3: How far the island is from the bank of the lagoon.
*Sara has a few groups restate the goal of the task.*
Sara: Does each group feel like they have a starting point? Ok, go for it.

In this lesson, Sara met the purpose for the practice of launching promoted in the professional development without using pressing and orienting moves. During her fourth and fifth lessons, she incorporated both orienting and pressing moves in her launches to support students in understanding the goal of the task and ensuring they could productively engage with the mathematics of the task. For example, as students conjectured possible approaches they may use to engage in the mathematics of the task in her fourth lesson, several suggested a mathematical approach that they learned about the previous year. Desiring for students to consider multiple ideas or strategies, Sara pressed the students, stating, “we are going to go beyond where you went last year – what can you bring to the table this year that you didn't have last year?” and “I would like for you to go beyond where you have gone in the past.” Similarly, in her fifth lesson students engaged in a task in which they worked to find the maximum area of a figure with a fixed perimeter. During the launch, as students suggested individual areas using a length and width they had chosen, Sara pressed students to consider multiple possibilities so they could engage in the task toward her mathematical goal by asking, “So where are you going to get the most space (area) and why – think about that question.”

In addition to her use of pressing moves, she used several orienting moves to link students’ approaches to visually representing the task together. During her launch in her fourth lesson Sara noticed that there were several representations across the groups in the class. She brought the class back together to conclude her launch and had the following exchange after a student (S1) shared her representation with the whole class:

Sara: Does anyone disagree with that picture?
S2: Aren't there two ziplines on the same side so both the towers have the same starting point?
S3: That's what I was thinking. They are both going to the same place
Sara: How does that go against S1's picture? (*orienting move*)
*Students debate two different representations*
Sara: Let's look at the problem again and see if anything in the problem helps clarify ...S2 says the tower should be on the same side. Anybody disagree with S2's diagram? (*orienting move*)

**Launching mathematics tasks – Brenda.** Similar to Sara, Brenda's purpose for launching in her first lesson was to get students working on the task as quickly as possible. She began by stating to the class that she wanted them to engage in doing "a math activity to show how math is used in real life." After this statement, she handed out a copy of the task to students and told the class to "work in their groups." In Brenda's second lesson, her purpose for launching was to support students in understanding the context of the task by asking fact-based questions to pull information from the text. Across the remainder of her lessons, Brenda's purpose for launching evolved to focus on providing opportunities for students to elaborate their thinking about the context and goal of the task while refraining from supporting students in developing possible mathematical approaches or ideas. For example, she had the following exchange in her fourth lesson where she refrained from eliciting possible mathematical approaches to engaging in the task during her launch.

Brenda: So, what is the question?...I heard a word over here – minimize. So, what does minimize mean?
S1: Smaller.
Brenda: And we want what?
S2: The smallest distance for the ziplines.
Brenda: We want the smallest amount of zipline. There are lots of ways to find the smallest amount of zipline. That is your challenge today.

**Summary of recompositions of the core practice of launching.** For Dawn, her launches evolved from withholding the publicizing of students' understanding of the context and goal of the task to providing opportunities for students to elaborate their thinking and supporting students in understanding both the context and goal of the task. This change was associated with her increased use of pressing and orienting moves. For Carla, all of her lessons included launches that ensured that students understood the context and mathematical goal of the task and were able to begin to engage with the mathematics of the task, and her highest quality enactments were predominately related to her use of orienting moves. For Brenda and Sara, they added the practice of launching to their second lessons and grew to support students in meeting the promoted purpose of launching in different ways. For Sara, her purpose progressed to focus on providing opportunities for students to elaborate their thinking and support students in understanding both the context and goal of the task, and her progression related to her increased use of orienting and pressing moves. For Brenda, her purpose for launching began by withholding the publicizing of students mathematical approaches and grew to ensure they could engage productively with the task, and for the most part did not include the use of orienting or pressing moves.

Dawn, Sara, and Carla used pressing and orienting moves to varied degrees but for similar goals, which aligned with their purpose of launching. Brenda did not use orienting or pressing moves in her launches. This evidence suggests that all four teachers learned to enact the core practice of launching toward the purpose promoted in the professional development. Yet the ways they enacted the practice to support student learning differed and related to their use of pressing and orienting moves. All four teachers recomposed this practice to a more ambitious purpose, but only some of them recomposed the practice to include instructional moves that were responsive to, and support of students' engagement in the task. These findings suggest that the quality of launching can improve when teachers use pressing and orienting moves that are in response to students' thinking.

**Monitoring Students Engagement in The Task**

For the four teachers, across their enactments of monitoring in their five lessons, they advanced to meet the purpose promoted in the professional development as shown in Table 4, which was to support students in engaging with the task in ways that advance toward the mathematical goal (see Appendix A for monitoring framework and Appendix C for rubrics). While their enactments of monitoring improved or were maintained, their use of pressing and orienting moves and their goals for the use of these moves differed as shown in Tables 5 and 6.

Table 4

Monitoring Engagement ARQ Scores – Meeting the Purpose of Monitoring

| Academically Relevant Questions Rubric (0 – 4) | | | | |
|---|---|---|---|---|
| **Enactment** | **Dawn** | **Carla** | **Sara** | **Brenda** |
| **1** | 2 | 4 | 4 | 4 |
| **2** | 4 | 4 | 3 | 4 |
| **3** | 4 | 4 | 4 | 4 |
| **4** | 4 | 4 | 4 | 4 |
| **5** | 4 | 4 | 4 | 4 |

Table 5

Monitoring Engagement – Pressing (TP) Scores

| Teacher Press / % Pressing Moves (# of Pressing Moves) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Enactment** | **Dawn** | | **Carla** | | **Sara** | | **Brenda** | |
| **1** | 2 | 7% (4) | 3 | 12% (21) | 2 | 6% (11) | 1 | 6% (9) |
| **2** | 3 | 10% (8) | 4 | 8% (14) | 3 | 5% (14) | 1 | 5% (11) |
| **3** | 4 | 17% (37) | 3 | 11% (12) | 3 | 9% (9) | 1 | 6% (16) |
| **4** | 4 | 9% (21) | 4 | 8% (10) | 4 | 12% (25) | 4 | 8% (18) |
| **5** | 3 | 9% (9) | 3 | 10% (11) | 4 | 16% (26) | 4 | 5% (15) |

Table 6

Monitoring Engagement – Orienting (TO) Scores

| Teacher Orienting / % Orienting Moves (# of Orienting Moves) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Enactment** | **Dawn** | | **Carla** | | **Sara** | | **Brenda** | |
| **1** | 0 | 0% (0) | 3 | 4% (8) | 0 | 0% (0) | 0 | 0% (0) |
| **2** | 1 | 5% (4) | 2 | 8% (13) | 2 | 5% (14) | 0 | 1% (2) |
| **3** | 2 | 0% (1) | 4 | 8% (9) | 2 | 3% (3) | 0 | 0% (0) |
| **4** | 4 | 9% (21) | 4 | 15% (19) | 2 | 3% (6) | 1 | 2% (4) |
| **5** | 4 | 20% (9) | 2 | 6% (7) | 2 | 3% (5) | 3 | 3% (10) |

For Dawn, she used pressing moves throughout her lessons during monitoring, and over time incorporated more orienting moves, as seen in her fourth and fifth lessons. Across her lessons the degree to which these moves were supportive of students’ thinking seemed to relate to the quantity of moves she used. For Carla, she consistently used both pressing and orienting moves during monitoring, and these moves were most often supportive of students’ thinking. For Sara, she progressed to use an increased percentage of pressing moves to support students’ thinking and used a small number of orienting moves. Finally, for Brenda, she used pressing moves across all of her lessons and her moves became more supportive of students’ thinking. In addition, she began to use orienting moves in her later lessons and in her last lesson these moves were more supportive.

**Monitoring student’s engagement in the task – Dawn.** In Dawn’s first lesson, her overall purpose for the practice of monitoring was to understand students’ thinking and provide them with opportunities to refine their thinking independently or with their peers. Throughout this lesson, she approached small groups as they engaged with the task, probed students’ thinking related to their strategies or questions, and then left them to discover on their own without responding to their mathematical thinking while she attended other groups. As she monitored , she pressed students four different times to continue engaging with the task with statements such as, “How can you experiment with your number theories to figure out how to minimize that?” and “How can you manipulate your numbers here to come up with some possible distances?” to support them in working to develop an understanding of minimization. After each of these attempts, she left the group, and in subsequent conversations with them did not further explore their thinking related to this mathematical idea.

As Dawn's practice of monitoring progressed, her purpose for the practice shifted from leaving students to explore on their own, to supporting students in persisting with their mathematical conjectures about the task toward her learning goal. Corresponding to this shift in purpose, Dawn used an increased percentage of pressing moves across her second and third lesson. In her second, her goal for pressing moves during monitoring continued to be to press students to continue engaging with the task and then leave the group. In her third, she used pressing moves during conversations with small groups to support them in working toward her learning goal for the lesson, which was to find a way to generalize the sum of an arithmetic sequence. For example, as she approached a group, a student stated that they had the answer for the sum of the first four terms:

S1: We got $28.
Dawn: How did you get it?
S2: We added the numbers, plus 4, plus 6, plus 8, plus 10.
Dawn: You added it up. There is no shame in that game, if that is working for you and you understand it, then you are showing me some of your mathematical knowledge. However, let's take what you have been doing and instead of saying how many days does it take her to get to $700, what if I said she is going to work 60 days or 100 days. Are you going to build that table out all the way down to 100 days or would we like to come up with a better way? (*pressing move)*
S2: Maybe an equation would be a better way.
S3: What if we did 2(n+1)
*Student explains what their generalization represents in the context of the problem and Dawn probes them to understand their thinking*
Dawn: What does that part of the equation tell you? What is that output value telling you?
S3: How much they get per day.
Dawn: Is that the same thing as how much they've made after working 100 days?
S1: No, that is what I am trying to figure out.

Dawn: Why don't you explore that idea of a table and see if you can find an equation that way? (*pressing move)*

Rather than leaving the group with a pressing move, Dawn engaged with the group of students to understand their strategy, used a pressing move in response to their strategy aimed toward her mathematical goal, and worked to support them in building a way to generalize the sum of the sequence.

In addition to using pressing moves to support students toward her learning goal, there was an increase in quantity and the quality of Dawn's uses of orienting moves. While she did use orienting moves in her second and third lessons during monitoring, they were few in number and were used to redirect distracted students to keep them engaged. In her fourth and fifth lessons, she used orienting moves to draw attention to a student's mathematical strategy so that others could relate the strategy to their own thinking and engage productively in the mathematics. In the following abbreviated example taken from Dawn's fourth lesson, students engaged in the Zipline Task. Dawn approached a group who had been working individually to construct visual representations of the problem based on their understandings of the context.

Dawn: S1, whatcha got? (*S1 explains their representation*)
Dawn: Oh – I like that, but what is this representing? (*S1 explains*)
Dawn: Where is your island (*the movable point*)? Look at S2's triangles. (*orienting move*) S3, your diagram looks good too.
Dawn: (*pointing to S4's visual representation*). Look right here, see how she has hers' drawn. Can you (S4) explain to her (S1) why you put that there? (*orienting move*)
*S4 explains to S1...Dawn supports other groups and comes back to this group*
S3: Am I doing this right?
Dawn: You are, I just want you to be very careful with the way you are labeling. I think if you look right here (pointing to S3's work) and you look at S4's

diagram as well, can both of these look like this based on her (S4) diagram? (*orienting move*)

*Dawn walks away, engages with other groups, and then comes back.*

S3: Ok, so this one would not be 600 correct?

Dawn: Why not? (*S3 explains their reasoning*) Alright, let's look at S2's diagram to support what you are doing already. (*orienting move*)

Dawn: (*Dawn uses S2's representation to connect to S3's representation*) S3, I need you to tell S2 what you are doing so he can figure out how your work can align to his diagram. (*orienting move*)

In this example, after she conversed with one of the students about their struggles to visually represent the problem, Dawn oriented students to one another's representations with a goal of developing a common representation so students could progress the lesson toward her mathematical learning goal.

**Monitoring student's engagement in the task – Carla.** For Carla, across all five of her lessons, her purpose for enacting the practice of monitoring was to support students in investigating their mathematical conjectures and formalizing their mathematical ideas. In the following example from Carla's fourth lesson, students engaged in the Zipline Task. As students worked in small groups to determine the lengths of the wire relative to one location of the movable point, Carla used pressing and orienting moves to advance them toward her learning goal.

Carla: So, what did you guys come up with?

S1: We did the Pythagorean theorem for half of the 600 (*placing the point halfway between the two towers*).

Carla: How can we tell if this is going to be the best place to put the island (the movable point)?

*Students debate where to place the point to minimize the length of the wire.*

Carla: Is there a way to mathematically prove what you just said? (*pressing move*)

S3: So, could we pick a place close to the 100-meter tower?

S4: I think it should be in the middle.
Carla: So, S4 is thinking that if it is exactly in the middle it should be less zip line. How can you prove or disprove what she is saying? (*pressing move*) *S4 continues to conjecture that having the point in the middle will minimize the length of the wire.*
Carla: I want you (S4) to prove that. (S4: I can't.)
Carla: Well then we are going to ask for some help around the table. (*orienting move*). The best way to prove something is to see if you can disprove it. I want you to prove or disprove (pressing). Try moving your island (to S4). And you (S3) move your island somewhere else. And you (S1) and you (S2) move yours somewhere else. And then compare your numbers. (*orienting move*)

This example exemplifies the ways Carla worked to enact the practice of monitoring and the ways she used pressing and orienting moves in response to students' thinking across her five lessons. Throughout her lessons, during monitoring she used orienting moves so that students could work from a common understanding or representation and used pressing moves to support students in building from their conjectures toward her mathematical goal.

**Monitoring student's engagement in the task – Sara.** In her first lesson, Sara's purpose for the practice of monitoring was to support students in engaging with the task toward their own ideas about the mathematics of the task. To meet this purpose, Sara predominately used a pattern of probing students to understand their thinking and mathematical work and then explaining contextual features of the task and mathematical procedures so that students could continue to engage. While she used several pressing moves during monitoring in this lesson, they were typically too vague for students to understand the mathematical intent of the pressing move. For example, after students shared their mathematical thinking and conjectures about how to solve the problem, Sara used vague pressing moves such as, “you need to prove it” or “prove that it is a good idea” as she left groups of students. She did not use orienting moves during monitoring in this lesson.

As Sara’s lessons progressed, her purpose for monitoring evolved to support students in engaging more deeply with the mathematics of the task and ensuring that students engaged with each other’s mathematical thinking as they worked on the task toward a mathematical goal. Across her next four lessons, Sara grew to use an increased percentage of pressing moves, maintained use of a small percentage of orienting moves, and the quality of both of these moves became more focused on students’ mathematical thinking during monitoring. In her second lesson, Sara continued a pattern of probing and explaining. But, she also used orienting moves to support students in engaging with each other’s mathematical thinking as well as pressing moves to encourage students to try out their conjectures. In the following example, students engaged in the Zipline Task and conjectured about the relationship between the Pythagorean theorem and the distance formula.

Sara: What y’all got going on?
S1: The Pythagorean theorem.
Sara: Okay. What are you doing with it?
S1: This side is going to be less than this side because this is shorter so like it is less far to go.
S2: I have a question. If we could do the Pythagorean theorem with this could we do the distance formula?
Sara: Great question. What do you all think? (*orienting move*)
S3: I don’t think you can because...in my opinion...*student explains their conjecture*
Sara: That’s a great question, so is there any way we can position the situation on this graph paper so that we would be able to use the distance formula? (*pressing move*)

In her fourth and fifth lessons, the frequency of Sara's use of pressing moves increased and her use of pressing moves became more attentive to student's mathematics during monitoring, examples of which are shown in Figure 6. In each of these lessons, students were engaged in optimization problems to determine the minimum or maximum value of a composite rational function and a quadratic function, respectively. In both cases, students worked in groups to build tables and examine patterns. Sara's learning goal for each of these lessons was for students to develop a generalized function to represent their patterns and she used pressing moves to support students in extending their thinking toward her goal.

| | |
|---|---|
| 4th Enactment<br><br>Goal to minimize the length of the sum of two hypotenuses | • Can you find some way of representing the wire used? How can we involve this length into our function?<br>• How can we represent the amount of wire with the length from the tower to the movable point as my input?<br>• How are you going to check and see if it will give you the least amount of zip-line?<br>• What can you do to generalize what you are doing, instead of using specific numbers, can you take it into a general world?<br>• Can we think about how we can write an equation that represents the wire used? |
| 5th Enactment<br><br>Goal to maximize area of a rectangular figure | • Is there anything else we need to know about to figure out where the best rectangle is for our gold? ...You are continuing to make a table – to make that table what did you do every time?<br>• I know you are convinced, but can you prove it mathematically – with numbers or variables to prove it is the best one?<br>• I need you to ask yourself how you get something that represents the area. If I give you a L what do I do to get the W and then the A? What are you doing every time by writing it as an expression? |

Figure 6. Examples of Sara's Pressing Moves in Fourth and Fifth Lessons.

**Monitoring student's engagement in the task – Brenda.** Finally, for Brenda, in her first, second, and third lessons, her purpose for monitoring was to support students in engaging with the task and develop their own ideas about the mathematics of the task. Brenda predominately used a pattern of probing students to assess their understanding of the context of the task or their mathematical conjectures of the task and then explaining contextual features of the task and mathematical procedures so that students could continue to engage. While she used several pressing moves throughout these lessons, she used the same move each time to press students to extend their thinking. For example, in her first lesson, after students had engaged with the Zipline Task for an extended period of time, they had developed a way to find the length of the wire when the point was in the middle between the towers. She went around to each group and posed the question, "If I shift the point, what happens to the length of the wires?" Her goal for this pressing move was for students to consider other locations and make progress in determining the location of the point that minimized the length. For the most part, she did not use orienting moves during these enactments.

In her fourth and fifth lessons, the frequency of Brenda's use of pressing moves remained consistent during monitoring. However, the quality of her pressing moves improved to be more responsive to, and supportive of, students' mathematical thinking. In her fourth lesson, rather than simply pressing students to shift the island as she did in the previous example, she supported them in understanding the task as a means to help students make progress with the task. For example, after a group struggled to consider other locations for the point, Brenda had them reread the problem:

Brenda: Does it say it is in the middle of the lagoon?
S1: No, it just says in.
Brenda: You figured out for right here (*pointing to the middle)*...But we just said it might not be in the middle. What would happen to those lengths if the point was over here?
S2: This one (base) would be smaller and this one (another base) would be bigger.
Brenda: Right, so you need to calculate some more possibilities. Is that the only way the point could go?
S3: No, it could go the other way.
Brenda: So, you need to make a table and see what happens when you go either way. (*pressing move*)...*Brenda goes to other groups and then comes back*
Brenda: So, what did you find out, did it go down?
S2: It went up.
S4: Is that bad?
Brenda: Well, I am trying to minimize...so if you went one way and it went up, maybe you should check the other way. If they both go up, does that mean the middle is absolutely the minimum? (*pressing move*)

Rather than pressing and then leaving, Brenda pressed the group, responded to their understanding of the task, and support them in refining their mathematical approach. In addition, Brenda added orienting moves to her lesson during monitoring and the quality of these moves improved in her fifth enactment, as shown in Figure 7.

| 5th Enactment | See if you can do that with all three of those and see if you can help him understand. |
|---|---|
| Goal was to build a polynomial function | One thing that you are not thinking about is, he has it a little bit differently...<br>Think about how we could solve that – just think about it – help him understand what these pieces mean and help him solve it.<br>Think you could explain that to him? Why don't you let him explain – he has an equation – see if he can explain the equation to you. |

Figure 7. Examples of Brenda's Orienting Moves in Fifth Lesson.

**Summary of recompositions of the core practice of monitoring.** For Dawn, her purpose for monitoring evolved from leaving students to explore on their own to supporting students to persist with their mathematical conjectures and relating students' strategies together. This evolution of purpose corresponded to her use of pressing and orienting moves that were responsive to students' thinking. For Carla, all of her enactments of monitoring used pressing and orienting moves to support students in investigating their mathematical conjectures and formalizing their mathematical ideas.

Initially, Sara and Brenda had a similar purpose for monitoring, which was to support students as they used their own ideas to engage with the task. Both predominantly used probing and explaining moves, and a few pressing moves. For Sara, the purpose of monitoring changed incrementally and at the conclusion of these lessons was best described as supporting students to engage more deeply with the mathematics of the task and ensuring that students engaged with each other's mathematical thinking. This shift in purpose was accompanied by an increased use of pressing moves and goals for pressing and orienting moves that were more focused on responding to student's mathematical thinking. For Brenda, her initial purpose of monitoring remained unchanged until her fourth lesson, where her purpose shifted to being more responsive to students' mathematical thinking. This shift was supported by orienting and pressing moves with goals that were responsive to students' understanding of the task.

To varied degrees, all teachers used pressing and orienting moves to support students in engaging with the mathematics of the task during monitoring. This suggests that while teachers can enact the practice of monitoring with a purpose of supporting student engagement in mathematics, the use of pressing and orienting moves that are specific and responsive to students' thinking improves the quality of monitoring and fosters opportunities for students to develop their mathematical understanding.

**Facilitating Whole Class Mathematics Discussions**

Across all four teachers, their whole class discussions developed in different ways toward meeting the purpose of discussions promoted in the professional development (Table 7) – which was to facilitate a discussion of multiple students' mathematical work in relation to the mathematical goal of the lesson (Appendix A, C). In addition, across their lessons, their use of pressing and orienting moves differed as shown in Tables 8 and 9. Before summarizing teachers' discussions, it is important to note that for each teacher, these five lessons represent their attempts at delivering whole lessons that included all three core practices of launching, monitoring, and discussing. As shown in Table 7, at various times, each teacher did not incorporate a whole class discussion into their lesson, thus these summaries only include the discussions that were enacted during that instructional day. Any attempts that teachers made in subsequent days to continue working on the task are not a part of these data.

For Dawn, her discussions progressed to both meet the purpose promoted in the professional development and use pressing and orienting moves. For Carla, each of her discussions met the promoted purpose and she sparingly used pressing and orienting moves; however, when used, they were supportive of students' thinking. For Sara, she did not enact a discussion until her fourth lesson and during these discussion used both pressing and orienting moves that were somewhat supportive of students' thinking.

Finally, for Brenda, her discussions progressed to meet the promoted purpose and she rarely used orienting or pressing moves.

Table 7

Discussing Students' Mathematic – Meeting the Purpose of Discussing

| IQA | | | | |
|---|---|---|---|---|
| **Enactment** | **Dawn** | **Carla** | **Sara** | **Brenda** |
| **1** | 2 | 4 | | 2 |
| **2** | 2 | 4 | | 1 |
| **3** | 4 | 4 | | 4 |
| **4** | | 4 | 4 | 3 |
| **5** | 4 | | 4 | |

Table 8

Discussing Students' Mathematic – Pressing (TP) Scores

| Teacher Press / % Pressing Moves (# of Pressing Moves) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Enactment** | **Dawn** | | **Carla** | | **Sara** | | **Brenda** | |
| **1** | 2 | 8% (1) | 2 | 3% (2) | | | 0 | 0% (0) |
| **2** | 2 | 18% (2) | 3 | 3% (3) | | | 0 | 0% (0) |
| **3** | 2 | 7% (4) | 2 | 2% (1) | | | 2 | 2% (2) |
| **4** | | | 3 | 6% (6) | 2 | 4% (5) | 0 | 0% (0) |
| **5** | 4 | 12% (13) | | | 2 | 8% (7) | | |

Table 9

Discussing Students' Mathematic – Orienting (TO) Scores

| Teacher Orienting / % Orienting Moves (# of Orienting Moves) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Enactment** | **Dawn** | | **Carla** | | **Sara** | | **Brenda** | |
| **1** | 0 | 0% (0) | 3 | 5% (3) | | | 0 | 0% (0) |
| **2** | 2 | 17% (3) | 2 | 3% (3) | | | 0 | 0% (0) |
| **3** | 3 | 3% (2) | 2 | 5% (3) | | | 2 | 3% (4) |
| **4** | | | 0 | 0% (0) | 3 | 7% (8) | 2 | 7% (2) |
| **5** | 3 | 8% (9) | | | 3 | 6% (5) | | |

**Facilitating whole class mathematics discussions – Dawn.** In Dawn's first lesson, her purpose for discussing was to have a student share an approach and elicit students' mathematical answers. In this lesson, she had one student come to the front and share their work with the class. As the student shared their approach and answer, Dawn repeatedly asked other students in the class how their answer compared to the one shared.

Dawn's second discussion was similar in form, with a small shift in purpose away from the answer toward a focus on the mathematical goal of the task. In this discussion, students explored the Zipline Task. Rather than eliciting answers, after students shared she pressed the class toward her goal, stating, "the question we kept coming back to was, is putting it in the middle really the best place.*"* The class then had a short discussion to conclude the lesson about the fact that the middle is not necessarily the place that minimizes the length of the wire.

In Dawn's third and fifth lessons, her purpose for discussing shifted to focus on sequencing students approaches to build toward her mathematical goal. Across these two enactments, her discussions grew to include pressing and orienting moves that built from students' mathematical thinking. For example, in her third lesson students engaged in a task exploring how to generalize the sum of an arithmetic sequence. Dawn's goal was for students to explore the sum of an arithmetic sequence from a quadratic perspective and she had different students share approaches ordered toward her learning goal. As the discussion progressed, she used a pressing move to encourage the class to consider the need to generalize toward an expression, stating, "so that's great, but what if she kept working for 100 days, how much money would she have in 100 days?" Similarly, she used another pressing move for students to consider what this generalization might be if it was not linear, "So if it isn't a constant rate of change, what does that tell you about the type of equation you may or may not have?" After several groups shared their graphical representations, Dawn used a series of orienting moves to bring these approaches together for students to consider.

> Dawn: S1, can you come up and sketch your graph for me.
> *S1 comes up and sketches a quadratic function...*
> Dawn: S2 earlier had drawn a graph. What he graphed was the connection between the day he was on and the money he made for that day to get a linear function which is true for that connection. But what we wanted was a connection between the day and the sum total. S1, what did you find out when you graphed that? (*orienting move*)
> S1: "slopey."
> Dawn: Alright, so she got something slopey. If we go back over to S3's graph that they tried, an exponential graph, it looks kind of slopey too doesn't it. But what they figured out was that any time they tried to figure out an exponential function to support the data they knew was right, they couldn't. (*orienting move*)

**Facilitating whole class mathematics discussions – Carla.** Across Carla's first three lessons, her purpose for discussing was to facilitate the sharing of students' approaches to explain the procedures of their approaches toward her mathematical goal. In these discussions, Carla had several students share and predominately used a pattern of probing students to understand their approach and then explaining to the whole class to clarify what students did. As she transitioned between approaches, she used orienting moves such as "see if you used the same idea or something different", "how did this compare with the numbers your group had," and "compare it with your drawing" to ensure that all students were relating what the student was sharing to their own work.

Carla used limited pressing moves throughout these discussions, however, when she did use them they were to leverage and build upon students' thinking and approaches to support students in moving toward her learning goal. For example, pressing moves such as "Can we verify that this uses the least amount of wire?," "How do we know this is the best location for the point?," "What do you think would happen if we put the point right here?," and "How can you tell it is not linear?" were used throughout these discussions so that the whole class could together develop an understanding of her learning goal.

In Carla's fourth lesson, her purpose for discussing was to facilitate the sharing of students' answers to construct a common representation of the problem, and progress the discussion toward her mathematical goal. In this lesson, students explored the Zipline Task. In the discussion, rather than sequence the sharing of approaches, Carla chose to call on students to share their answers for different locations of the point, and as they did so, she used pressing moves such as, "How do we know if this is the best place to be or not?," "How does this one compare to the original?," " How can we set up a table that is going to give us some ideas?," and "You think it might be quadratic, how could we verify that?". These moves supported students in relating the previous length of the wire to the new length, building toward the mathematical goal of understanding minimization, and conjecturing about the type of function that could model the situation.

**Facilitating whole class mathematics discussions – Sara.** Sara did not get to a discussion in her first three lessons. In her fourth and fifth lesson, her purpose for discussing was to sequence students' approaches and then engage the class in understanding each approach toward her mathematical goal. In these discussions, Sara used a pattern of having a student come to the front to present their work, probing the class to understand the student's thinking, and explaining to clarify the student's contributions. Throughout these discussions she used pressing and orienting moves.

Across the two discussions, Sara's goal for using orienting moves was to focus the class's attention on a particular aspects of a student's mathematics. Statements such as "Can someone interpret S1's function?," "Does anybody see any problems with her notation here?," "Does this function represent what you did?," and "Look at his table and let's talk about what you all think about it?," represent orienting moves Sara used to draw on students' mathematics to progress toward her learning goal. While Sara used a similar number of pressing moves, these moves tended to be superficial or focused on pressing students to use a mathematical procedure, rather than to justify or extend their reasoning. Statements such as "Can someone make this like we are used to seeing in function notation?" and "Can we write this (linear function) differently but so that it means the same thing?" represent the ways Sara used pressing moves in both of these discussions.

**Facilitating whole class mathematics discussions – Brenda.** Across Brenda's first two lessons, her purpose for discussing was to encourage students to share the answers they obtained as they solved the task. In the following abbreviated example from her second discussion, students engaged in the Zipline Task and shared their totals for the length of the wire given a specific location they had tried.

Brenda: What did you come up with your smallest total?
S1: 622.1
Brenda: Anyone come up with anything smaller? You did – what did you come up with?
S2: 620.9

Brenda: What did you come up with?
S3: 621.1
Brenda: 621.1 you got 620.9 we are within 2/10ths.
*Discussion concludes with a conversation about the problem-solving process rather than the mathematics of the task.*

In Brenda's third and fourth lesson, the purpose for her discussions shifted from sharing answers to eliciting students' mathematical thinking and focusing on mathematical procedures. In both these lessons she had several students share and used a pattern of probing and explaining, in addition to a few orienting moves. In the following abbreviated example from her fourth discussion, students engaged in the Zipline Task.

Brenda: How many got an answer for the center of the lagoon? What did you get?
S1: 622.16
Brenda: Did we decide that was the best place?
S2: No
Brenda: So, if it is not in the middle of the lagoon, what do we need to do?
S3: Move it one ways or the other.
Brenda: And if we create tables what are we looking for?
*Brenda has a student share their approach and explains to the class how the student developed a function to represent the length of wire with respect to the location of the point*
Brenda: How do we create an equation when we don't know something, what do we use?
S2: A variable.
Brenda: A variable because that is how we know something will change. We use a variable and make this side x...If we know the total and this is x. How will we find this side?
S2: 600 – x
Brenda: So, then we were looking at possibly putting those into the formula to come up with a function that would tell us what that point was.

In this example, Brenda began her discussion by acquiring for an answer and progressed to focus on building the procedures needed to develop a function to represent the problem by using probing and explaining moves. Across her discussion, Brenda used pressing and orienting moves sparingly. However, she used pressing moves such as, “What can we do with the factors to create the quadratic?” to focus on the mathematics of the task and orienting moves such as, “Is that close to anybody else’s equation?” and “another group did this just a little bit differently” to attempt to relate across students’ approaches.

**Summary of recompositions the core practice of discussing.** For Dawn, her purpose for discussing advanced from focusing on sharing answers to sequencing students’ approaches toward her mathematical goal. In addition, she added the use of pressing and orienting moves to build on students’ mathematical thinking and connect students’ approaches. For Carla, her purpose for discussing was to facilitate the sharing of students’ approaches in ways that shifted from explaining mathematical procedures to focusing on constructing a common representation to facilitate mathematics discussions toward her goal. In her discussions, she used orienting moves to transition between students’ approaches and used pressing moves to build upon students’ thinking.

For Sara, when she had a discussion in her fourth and fifth lessons, her purpose was to sequence students’ approaches and support students in understanding each approach toward her goal. During these discussions, she used orienting moves to focus on specific aspects of students’ mathematics and pressing moves to focus on procedures as she worked toward her learning goal. For Brenda, her purpose for discussing shifted from having students share their answers, to eliciting students’ thinking to focus on the procedures of the mathematics the task. During these discussions, she occasionally pressed students to focus on mathematics and made attempts at orienting students to one another’s approaches.

All four teachers, to varied degrees, were able to enact the core practice of discussing toward a purpose that sought to facilitate multiple students' mathematical approaches toward their learning goal. Across these discussions, those that were of high quality included the use of pressing moves to focus on the mathematics of students' approaches and orienting moves that made connections between students' approaches. This suggests that while teachers can enact the core practice of discussing with a purpose to facilitate between students' approaches, the inclusion of pressing and orienting moves that are focused on connecting the mathematics of these approaches improves the quality of discussions and fosters opportunities for students to develop their mathematical understanding toward teachers' learning goals.

**Summary of Findings**

This study was designed to address the question: *In what ways, and to what extent, did teachers recompose the core practices of launching, monitoring, and discussing, over the course of two implementations of professional development?* First, I asked how and in what ways teachers enacted the practices of launching, monitoring, and discussing across their attempts at ambitious mathematics teaching. Second, I asked how and in what ways teachers recomposed these practices to use pressing and orienting moves across their enactments. Results suggest that all four teachers entered the professional development with practices of varying quality enacted for different purposes and in different ways. Over time, they recomposed each practice in ways that were more closely aligned with ambitious mathematics teaching.

For the practice of launching, all teacher's progressed to support students in understanding the context and goal of the task. Three of the teachers, Dawn, Carla, and Sara, recomposed their practice of launching over time to support students in considering multiple approaches, ideas, or strategies prior to working to solve the task. To do so, they used a select number of orienting or pressing moves, indicating that a small change in the responsive moves teachers use in launching can yield larger changes in the overall quality of the core practice of launching.

For the practice of monitoring, all teacher's progressed to support students in engaging with the task and making progress toward its mathematical goal. Their enactments were marked by pressing moves to support students in investigating or persisting with their mathematical conjectures or in formalizing their mathematical ideas. Dawn and Carla recomposed their practice of monitoring to ensure that students engaged with each other's mathematical thinking by using orienting moves. These results suggest that as teachers monitor students' engagement in mathematics, the inclusion of pressing and orienting moves that are specific and responsive to students' thinking can improve the quality of the practice and foster opportunities for students to collectively develop their mathematical understanding.

For the practice of leading a discussion, teachers' purposes progressed in two different ways. Dawn and Sara recomposed their practice of facilitating discussions with a purpose of sequencing students' approaches to meet their learning goal for the lesson. To meet her learning goal for the lesson, Sara used orienting moves to focus on specific aspects of student's mathematics and pressing moves to focus on procedures. Dawn used pressing and orienting moves together to build from students' mathematical thinking and connect students approaches.

Carla and Brenda enacted their discussions with a purpose of eliciting students' thinking in order to focus on developing mathematical procedures. Brenda used few pressing or orienting moves, and her goals for these moves aligned with this purpose. Carla used orienting moves to transition between students' approaches and used pressing moves to build upon students' thinking to support them in understanding procedures.

These results suggest that as teachers facilitate mathematics discussions, their purpose for the practice works together with their goals for the use of orienting and pressing moves. When both the purpose of the practice and the goals for moves are aligned toward ambitious teaching, as in the case of Dawn and Sara, enactments are of higher quality. When the purpose of the practice is not aligned with the goals for instructional moves, the effects of these moves can still improve the quality of a practice that was not closely aligned with an ambitious purpose, as was the case with Carla's purpose for leading discussions focused on procedures and her use of and goals for pressing and orienting moves that focused on connecting the mathematics of students' approaches.

## Discussion

The detailed analysis supported by a framework of hierarchical modularity (Simon, 1973) and situated learning theory (Wenger, 1998), allowed for an in-depth investigation of the ways teachers recompose different aspects of their practice. The framework highlighted both the ways teachers bring together imagination and alignment in their lessons and a conceptualization of practice that manages the complexity of ambitious teaching and attends to multiple practices. My attention to changes in practice at the subsystem levels of large grain-size core practices and smaller grain-size core practices as instructional moves, along with the purposes and goals for enacting these practice and moves, provides empirical evidence that relations within and across practices of teaching and their effects on the quality of instruction is complex.

The overarching question of this study addressed the ways and extent to which teachers recomposed core practices for ambitious teaching. Applying my theoretical frame to attend to the complexities of ambitious teaching, I presented results from teachers' enactments of launching, monitoring, and discussing as core practices of teaching a task-based lesson. In addition, I presented results of teachers' uses of pressing and orienting moves nested within three larger practices as two actions that can support students in productively engaging in mathematics tasks in ways that further their conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions toward mathematics. Across both of these levels of practice, I also presented findings related to the ways teachers developed both their skill in enacting practices and their aims for enacting them.

Results indicated that across the three core practices, teachers were able to recompose their practices with more ambitious purposes. In some cases, teachers' uses of pressing and orienting moves with different goals, although few in number or percentage, lead to larger changes within each practice. In other cases, even when the purpose for enacting a practice was not ambitious, the use of pressing or orienting moves that focused on students' mathematical thinking can also lead to improvements in the quality of a practice. These findings strengthen the argument that positive changes in smaller grain-sized practices can support improvements in larger-grain sized practices.

Results also indicated that some teachers were able to recompose their teaching by adding new core practices and, over time, purposing them for ambitious teaching. The fact that Sara and Brenda quickly repurposed their practice of launching toward the purpose promoted in the professional development suggests that teachers may be able to add the practice of launching more quickly than they can add the practice of discussing. This is not surprising given the successive and contingent nature of these practices. Monitoring students or facilitating a discussion presupposes students have started to work productively on an instructional task. Of the three focal core practices, launching is focused more on the context of the task and preparing students to engage with mathematics, and thus may not require the use of as many responsive moves.

These data also suggest that facilitating mathematics discussions may be more difficult for teachers to learn to enact. Again, this is not surprising given that this practice is more contingent on the enactments of the previous practices and requires both attention to and knowledge of students' thinking. Future studies could explore the ways teachers work to recompose practices of the same level together using a hierarchical perspective to attend to the teaching of whole lessons. In a complementary paper, I have begun this work by considering the requirements of an analysis plan to examine the process of recomposing practices of launching, monitoring, and discussing and provided examples from this study to support the potential of this approach (Webb, in preparation a).

Like other studies, these findings suggest that a core practice approach is a productive way to both design for and study teacher learning of ambitious teaching. This study extends the emerging knowledge base about core practices to include a framework that addresses calls to more clearly conceptualizes the complex work of ambitious mathematics teaching (Clarke & Hollingsworth, 2002; Opfer & Pedder, 2011) , attends to multiple core practices and teachers' aims for enacting them (Jacobs & Spangler, 2017; Jansen et al., 2015), and may further reconcile calls to relate teacher learning across settings (Kazemi & Hubbard, 2008; Sztajn, Borko, & Smith, 2017). While using a hierarchical modular approach was productive for this study, I recommend caution in using this approach. From one perspective, a modular approach can seem reductive and ignore the complex social nature of practice, thus if used alone would be cause for concern in research on teacher learning of practice. In this study, I attempted to bring this framework together with a social theory of learning to ensure that, while attention was given to individual aspects of a complex system, the broader context of the study was situated within a commitment to practice as a social endeavor and a respect for teachers' existing practice.

Mathematics teaching is complex, and there is much more to teaching than a subset of practices. In this study, I suspended attention to some of these complexities. My focus on three large grain-sized core practices and two instructional moves reduced my data to support my research questions, methods, and interpretation of my findings. By doing so, there are a variety of details that are not considered in this analysis, including other core practices of varied grain size such as checking for understanding, providing feedback on student work, and noticing, among others. This study also failed to attend to non-interactive core practices, such as designing lessons or setting learning goals, and interactive practices, such as classroom management, fostering student agency, attending to issues of race, or implementing norms that are not necessarily subject specific. And finally, this study analyzed five lessons across two iterations of a professional development and should not be seen as representative of teachers' daily practice. Changes in their practice should be recognized within the context of this timeframe, and teachers' commitment to attempting to enacting these core practices across their five lessons as part of their participation in the professional development.

While this study has these limitations, a strength of bringing together a conceptual model of teaching that uses hierarchical modularity with social nature of learning is that it both aligns with, and motivates the need for further studies to strengthen the research base on teacher learning of practice across settings. My focus on using hierarchical modularity and engagement, imagination, and alignment offers a way to analyze learning of core practices across the settings of professional development and classroom practice. In addition, it furthers the field in finding common approaches to investigating both prospective and experienced teacher learning. For prospective teachers, who come to learn about the practices of ambitious teaching with limited conceptions of practice, this conceptual model provides a useful starting place. For experienced teachers, this conceptual model respects their existing practice while also providing a way to investigate it and consider other possibilities for practice.

In this study, I brought together imagination and alignment to analyze the work individuals do to as they try out new possibilities for practice and reconcile them with their existing practice. From a perspective of engagement and alignment, other studies have explored teacher learning in professional development to examine the ways teachers bring their own perspective of students and teaching into professional development and negotiate meaning of practice over time (Sztajn, Wilson, Edgington, Myers, and Partner Teachers, 2015). With calls to relate the work teachers do in professional development to changes in their classroom practice (Jansen et al., 2015; Kazemi & Hubbard, 2008), future research could explore the ways that teachers bring together engagement and imagination in rehearsals purposed for ambitious teaching to reify the meaning they are making, imagine new possibilities for themselves, and relate this imaginative work to their classroom practice. Studies such as these would support the field in developing a better understanding of the design and study of professional development and the coevolution of teachers' participation in engagement, imagination, and alignment across settings.

# CHAPTER IV

# DESIGNING REHEARSALS FOR SECONDARY MATHEMATICS TEACHERS

There is consensus among mathematics teacher educators (MTEs) that students learn meaningful mathematics when they collaboratively engage in mathematical activity in dialogic, learner-centered environments as teachers elicit and use their thinking to guide instruction (e.g., Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Kilpatrick, Swafford, & Findell, 2001; Munter, Stein, & Smith, 2015; Webb et al., 2014). Often characterized as ambitious (Anthony & Hunter, 2013; Jackson & Cobb, 2010; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010), instruction of this form has positive implications for student learning (Boaler & Staples, 2008; Franke, Webb, Chan, Ing, & Battey, 2009; Stigler & Hiebert, 2004; Tarr, Reys, Reys, Chavez, Shih, Osterlind, 2008). Recent progress by teacher educators and researchers has more clearly described core instructional practices that advance this vision of teaching (Ball & Forzani, 2009; Forzani, 2014; Jacobs & Spangler, 2017; McDonald, Kazmi, & Kavanagh, 2013; Windschitl, Thompson, Braaten, & Stroupe, 2012), and MTEs are exploring ways to support teachers using a core practice approach (Ball, Sleep, Boerst, & Bass, 2009; Boerst, Sleep, Ball, & Bass, 2011; Lampert, 2010; Lampert et al., 2010; Lampert, et al., 2013).

Drawing upon three pedagogies of practice proposed by Grossman and her colleagues (Grossman, Compton, Igra, Ronfeldt, Emily & Williamson, 2009), teacher educators often represent the work of ambitious teaching, decompose it for analysis and discussion, and design learning opportunities for teachers to approximate practices of ambitious teaching. One approximation that has grown in popularity is *rehearsal*. Rehearsals are experiences in which participants take on the role of teacher, student, or observer to rehearses practices central to ambitious teaching and receive in-the-moment feedback from a teacher educator, and are designed to be a space where teachers can deliberately try out the practices of ambitious teaching in less complex settings. Emerging research on the use of rehearsal suggests they support prospective teachers in understanding and learning to enact ambitious teaching practices (Boerst et al., 2011; Campbell & Elliot, 2015; Ghousseini & Herbst, 2014; Han & Paine, 2010; Hunter & Anthony, 2012; Kazemi, Ghousseini, Cunard, & Turrou, 2016; Kazemi & Wæge, 2015; Lampert et al., 2013).

In professional development (PD), teachers often engage in practice-based professional learning tasks structured around artifacts of teaching, such as students' written work (Kazemi & Franke, 2004), classroom video (van Es, Sherrin, 2006), or clinical interviews with students (Sherin, Jacobs, & Phillip, 2001) to make practices of teaching public for learning (Silver, Clark, Ghousseini, Charolambous, & Sealy, 2007; Wilson & Berne, 1999). Evidence suggests PD that relates new learning to teachers' existing practice influences their visions of teaching and can lead to changes in classroom practice (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Desimone, Porter, Garet, Yoon, Birman, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001; Goldsmith, Doerr, & Lewis, 2013; Cohen & Hill, 2001). Evidence also suggests that ambitious teaching is difficult to enact (Darling-Hammond & Synder, 2000; Lampert, 2010; Kennedy, 2005), and mathematics teaching in the U.S. is often teacher directed, focused on procedures, and marked with few opportunities for intellectual engagement (Munter, Stein, & Smith, 2015; Stigler & Hiebert, 2004; Wiess, Pasley, Smith, Banilower, & Heck, 2003). In contrast to practice-based approaches that provide opportunities for teachers to appropriate knowledge of practice from professional development back in their classrooms, there have been more recent calls to design learning tasks for teachers that are situated within the in-the-moment complexities of practice to both problematize teachers existing practices and provide opportunities for them to engage in imaginative practice in less complex settings (Grossman & McDonald, 2008; McDonald, Kazemi, & Kavanagh, 2013; Kazemi & Hubbard, 2008; Sandoval, Kawasaki, Cournoyer, & Rodriguez, 2016).

In this paper, I share a design of rehearsals purposed for teachers in PD that builds from promising research findings on the use of rehearsals with prospective teachers. Beginning with a view of mathematics teacher PD as a boundary encounter (Sztajn et al., 2014; Wenger, 1998), I briefly outline a theoretical perspective of opportunities for teachers to imagine new possibilities for their practice through their engagement in rehearsals in PD. After a review of the literature on rehearsals in prospective teacher education, I describe a design of rehearsal for teachers in PD. I detail our design process for rehearsal for teachers and describe the ways we have worked with teachers in rehearsals. I then offer evidence relating teachers' participation in rehearsals with improvements in their practice over two implementations of participation in a PD. I conclude by identifying challenges and offering recommendations for other MTEs considering the use of rehearsals for teachers.

**Engagement and Imagination: Learning in Rehearsals in Professional Development**

Wenger (1998) introduced a theoretical perceptive for attending to social learning in community. From his perspective, knowledge is competence in a valued enterprise and knowing is "active participation in the practices of social communities and constructing identities" (p. 4) in relation to the enterprise. As a part of this theory, he presents the concept of boundary encounters. Boundary encounters (Wenger, 1998) represents an approach to theorizing the ways that different communities come together to learn from one another. During a boundary encounter, members of different communities bring elements of their practice into the boundary community. As participants engage together in activity they negotiate both collective and individual meaning. To further operationalize individual learning within and across boundaries, Wenger presents three "modes of belonging": engagement, imagination, and alignment.

*Engagement* is the sustained involvement in the negotiation of meaning in the boundary community through participation. *Imagination* denotes the images and connections individuals create to relate their own practice to the shared meanings of the boundary community and the ways they imagine new possibilities or alternatives for their own practice. *Alignment* represents the ways individuals coordinate their practice with the shared practices of the boundary community, their commitments to the practices of their other communities, and their own goals for themselves. Wenger (1998) emphasizes that combining engagement, imagination, and alignment in different ways brings into focus different opportunities to learn. These combinations are useful for exploring teacher learning in PD. In this paper, I use engagement and imagination to frame teachers' participation in rehearsals and to examine learning as teachers publicize in imaginative practice the meanings they are making of new possibilities for their own practice.

Over the last five years, as part of a larger research project investigating the design and implementation of a practice-based PD, our research team has worked to design rehearsals for secondary mathematics teachers and investigated the ways engaging in rehearsals of core practices can support teachers in enacting ambitious teaching practices. Elsewhere, I have shared the ways secondary mathematics teachers recomposed their practices of launching, monitoring, and discussing for more ambitious purposes and came to use instructional moves focused on students' mathematical thinking as they enacted these practices (Webb, in preparation b, Webb, in preparation c). In this paper, I share our work to design rehearsals as "a place where new ways of knowing can be realized" (Wenger, 1998, p. 215) when mathematics teachers take risks and explore alternatives for their existing practice.

## Supporting Prospective Teacher Learning in Rehearsals

Typically, a rehearsal is an interactive practice-teaching experience that takes place after prospective teachers have learned about specific practices of ambitious teaching, but before they have enacted these practices with students in whole class settings. In rehearsal, participants take on the role of teacher, student, or observer to rehearses practices central to ambitious teaching in a controlled environment and receive in-the-moment feedback from a teacher educator. Rehearsals are often conducted within a bounded “instructional activity” – a container that provides opportunities to engage in the practices and principles of ambitious teaching as well as develop mathematical knowledge for teaching ambitiously (Lampert & Graziani, 2009). In preparing prospective mathematics teachers, teacher educators may use instructional activities such as choral counting, contemplate then calculate, or number talks to provide teachers with opportunities to engage with multiple practices (Kazemi, Franke, & Lampert, 2009; Lampert et al, 2013) or instructional activities such as eliciting students’ reasoning or orchestrating a mathematics discussion to focus more closely on an individual practice (Boerst et al., 2011; Campbell & Elliott, 2015). Emerging research on rehearsals of core practices have demonstrated their potential to support prospective teachers in understanding the purpose of practices, learning to enacting these practices, and restructuring their understanding of the work entailed in beginning to teach with attention to students’ thinking (Boerst et al., 2011; Campbell & Elliot, 2015; Hunter & Anthony, 2012; Lampert et al., 2013; Tyminski, Zambak, Drake, & Land, 2014).

In the next sections, I outline our PD designed around cycles of investigation and rehearsals. Specifically, I share the principles and decisions that guided the design of rehearsals for the practices of *launching* a mathematics task, *monitoring* students’ engagement in the task, and *discussing* the task with the whole class toward a mathematical goal. I describe our design of rehearsals for teachers and offer evidence of the ways two teachers engagement in rehearsals related to changes in their classroom practice.

## Designing Rehearsals for Secondary Mathematics Teachers in PD

The design rationale and examples used in this paper were a part of a multi-year research project with a primary goal of understanding the ways a practice-based PD that included rehearsals could support secondary mathematics teachers in learning to enact ambitious practices in their classrooms. We began the project after reviewing the literature and piloting rehearsals with teachers throughout a semester-long doctoral seminar on design-based research in mathematics education (Design-Based Research Collaborative, 2003). Through four cycles of design and implementation, we have collected empirical support for this model and its potential to support teacher learning of ambitious teaching (Dugan & Jacobs, 2017; Webb, in preparation b; Webb, Wilson, Martin, & Duggan, 2015; Jessup, Webb, & Wilson, 2015). The work I share in this paper is from teachers' participation in rehearsals during the summer institute portion of two implementations of our PD during the 2015-2016 and 2016-2017 academic years.

### Practice-Based Professional Development

We began the project with a deep respect of the expertise that mathematics teachers bring to PD and the context of their daily work in classrooms with students. With the consensus view of effective PD (Darling-Hammond et al., 2009; Desimone, 2009; Elmore, 2002) as a foundation, we designed a 12-month professional development program consisting of a 60-hour summer institute followed by 20-hours of follow-up meetings throughout the school year. As designers, our team was guided by an assumption that an approach grounded in representations, decompositions, and approximations (Grossman et al., 2009) of ambitious teaching in practice-based ways could support teachers in relating their existing practices to those guided by students' mathematical thinking and promoted by the professional development .

We organized the summer institute around cycles of investigating core practices and collectively building a shared conception of practice. Our cycles of investigation began by representing and decomposing practice to identify and make public the core practices shown in Figure 8, for analysis and discussion. We then formalized the core practices of launching, monitoring, and discussing with frameworks organized around the purpose of the practice and potential goals for moves that teachers could use to support students as they enacted the practice (Appendix A). Using these frameworks, teachers then approximated these practices, first in practice-based ways (e.g., analyzing student work or classroom videos) and then in rehearsal. These approximations provided teachers with occasions to explore how they might repurpose their existing practices for more ambitious forms of teaching and experiment with new practices in rehearsals.

<table>
<tr><th>Core Practices of Focus in PD</th><th colspan="2">Practice-based approximation</th></tr>
<tr><td>1. Setting mathematical learning goals</td><td colspan="2">Explore learning goals in relation to tasks, standards, pacing, and knowledge of students</td></tr>
<tr><td>2. Selecting cognitively demanding mathematics task</td><td colspan="2">Assess and adapt tasks to meet learning goals and needs of students</td></tr>
<tr><td>3. Anticipating students’ mathematical thinking</td><td colspan="2">Anticipate ways students will engage with a task, their approaches, and potential barriers; watch videos of students engaging with tasks and debrief student’s work in relation to teachers’ anticipations</td></tr>
<tr><td></td><th>Practice-based approximation</th><th>Rehearsals</th></tr>
<tr><td>4. Launching cognitively demanding task</td><td>Anticipate barriers to student’s engagement with the task</td><td>Launching Rehearsal</td></tr>
<tr><td>5. Monitoring small group engagement in the mathematics of the task</td><td>Identify teacher moves, conjecture potential moves to use, and watch classroom videos</td><td>Monitoring Rehearsal</td></tr>
<tr><td>6. Selecting students’ mathematical work to share in whole-class discussion</td><td>Select samples of student’s work that lead toward a mathematics learning goal</td><td rowspan="3">Discussion Rehearsal</td></tr>
<tr><td>7. Sequencing selected work towards a learning goal</td><td>Order samples of student work toward a mathematics learning goal</td></tr>
<tr><td>8. Discussing the mathematics with the whole-class.</td><td>Discuss challenges of leading discussions and planning for discussion</td></tr>
</table>

Figure 8. Core Practices and Professional Learning Tasks.

**Design Principles and Teacher Learning Conjectures for Rehearsals**

Four principles guided our design of rehearsals for secondary mathematics teachers in PD and are summarized in Figure 9. The first two principles were derived from both research on rehearsals in prospective teacher education and our pilot research. The third design principle was informed by our pilot research and a commitment to relate the goals of the PD to the realities of secondary teachers’ instructional context. The fourth principle followed from our research focus related to teachers’ participation in rehearsals and enactments, and our commitment to respecting teachers' existing practice and personal goals.

| **Design Principles for Rehearsals** |
|---|
| 1) Rehearsals grounded in a *common task* reduce the mathematical complexities of rehearsals.<br>2) *Student profiles* centralized students' mathematical thinking and support teachers during rehearsal.<br>3) Rehearsing *sequential core practices* maintains a focus on teaching toward a learning goal.<br>4) *Feedback* from teacher educators should be focused on eliciting teachers' purpose for enacting the core practices and goals for their instructional moves. |

Figure 9. Design Principles for Rehearsals and Enactments.

First, drawing from literature on rehearsals designed for prospective teachers (e.g., Boerst et al., 2011; Lampert et al., 2013), results from our pilot work (Jessup, et al., 2015) confirmed that teachers' engagement in rehearsal was influenced by their familiarity with the mathematics of the task used in rehearsal. Thus, our design for each cycle of investigation began by engaging teachers in a mathematics task as we represented ambitious teaching and together with teachers, decomposed our own practice to identify the core practices of focus for discussion. We designed our launching, monitoring, and discussing rehearsals around one of these tasks to reduce the complexity of the mathematics so that teachers could engage in rehearsals with close attention to the purpose of each practice and goals for moves they could use .

Second, a central component of ambitious mathematics teaching is attending and responding to students' mathematical thinking (Franke, Webb, Chan, Ing, & Battey, 2009; Jacobs & Empson, 2016). We learned from our pilot study that while a common task was useful in maintaining a focus on the purpose of the practice in rehearsal, we needed a way to also support rehearsing teachers in focusing on students' mathematics so they could rehearse moves that were in response to what students might do during instruction. Thus, we designed what we called "student profile cards" to promote more authentic interactions around content and students during rehearsal. Each student profile was based on research on student learning and contained a hypothetical contextual or mathematical barrier, a mathematical approach a student might take while engaging in the task, and handwritten student work that represented the hypothetical students' thinking. These profiles served two related purposes. For the teacher taking on the role of a "student," the profiles supported them in playing the role of student more authentically with a description of the student's mathematical thinking, a record of mathematical work, and potential barriers a particular student may have when engaging in the task. For the rehearsing teaching, the profiles supported their rehearsal by providing records of mathematical work they could examine with the students in the moment. For each summer institute, we designed eight student profiles that were assigned to different teachers to use as they assumed the roles of students during rehearsals for each of the practices. Figure 10 provides two examples of student profiles used in the first implementation of the PD.

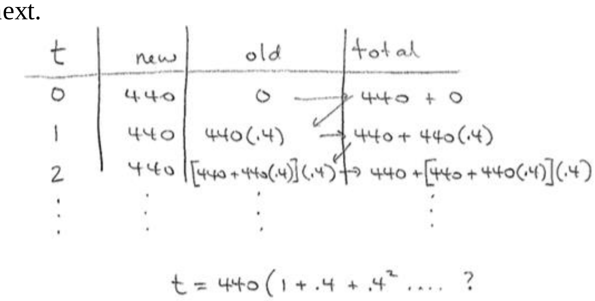
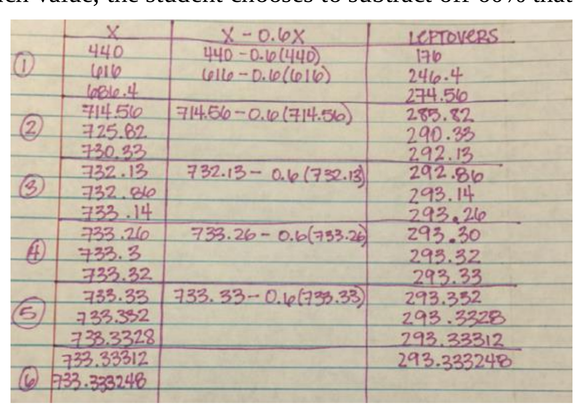
| Student Profile Examples | |
|---|---|
| The Task: A student strained her knee in an athletic competition. Her doctor has prescribed an anti-inflammatory drug to reduce the swelling. The student takes two 220mg tablets every 8 hours for 10 days. Her kidneys eliminate 60 percent of this drug from her body every 8 hours. How much of the drug is in her system after 10 days? If she continued to take the drug longer than 10 days, how much of the drug would be in her system? (adapted from NCTM, 2009) | |
| **Example #1** | |
| **Launching** | **Monitoring and Discussing** |
| *Barrier:*<br>Student misunderstands dosage rate (i.e. they want to begin at time = 0 and thus think she will take drug 4 times per day at 0, 8, 16, 24 hours, and then again at 0, even though 24 and 0 are the same time)<br><br>*Approach:*<br>Remembers something about geometric series and thinks this may work, but can't remember the formula so is going to make table with columns for new, old, and total to keep track and try to figure out the formula from the table. | *Mathematical Approach:* Student understands the problem and is able to create a table to represent the amount of medicine in the body at the beginning of the next dose (Total) and right before the next dose (Old). When computing each value, the student chooses to add 40% that remains rather than subtract the 60% that is eliminated. Additionally, they do not compute the values, but rather keep them in terms of the 440 mg dosage and the amount that remains (40%).<br><br>*Barrier:* Student sees the relationship between a "next, now approach" and thinks they can factor out the 440 and sees a pattern in what is not factored out. However, they are unable to move forward to generalize. They don't know what to do next.<br> |
| **Example #2** | |
| **Launching** | **Monitoring and Discussing** |
| *Barrier:*<br>Student misunderstands number of pills to be taken (i.e. is she taking 2 at a time, 1 every 4 hours, does it matter?<br><br>*Approach:* Student wants to create a table and look for a pattern. | *Mathematical Approach*: Student understands the problem and is able to create a table to represent the amount of medicine in the body at the beginning of the next dose (x) and right before the next dose (leftovers). When computing each value, the student chooses to subtract off 60% that is eliminated rather than add 40% that remains.<br><br>*Barrier*: Student does not see a recursive (next, now) pattern but is simply plugging in numbers each time.<br> |

Figure 10. Student Profile Examples.

Our third design principle was based on results from our pilot implementation and professional literature highlighting the complexity of a core practice approach and the challenge of attending to differing grain sizes of practices when designing for teacher learning (c.f., Jacobs & Spangler, 2017). In the first implementation, we designed a rehearsal in which teachers engaged with one “student” to simulate the smaller grain-size core practice of attending and responding to one students’ mathematical thinking. While teachers found this to be productive, they felt that this rehearsal was too fine-grained given the overarching goal of supporting teachers in learning to teach whole lessons that were more ambitious. However, teachers agreed that rehearsals grounded in small or whole group interactions with multiple students would best meet the goals of the PD, allowing them to rehearse meeting the purpose of the practices using moves that were responsive to more than one student (e.g., orienting moves).

In addition, teachers commented on the desire to engage in sequences of core practices with a variety of contextual factors, in particular under realistic time constraints. Thus, we learned that it was critical to choose core practices that were broad enough to be useful when teaching whole lessons and that core practices that build upon one another in a sequential manner across a lesson aligned well with the context of secondary mathematics teachers work. Subsequently, we revised the design to focus on the three large grain-size practices of launching, monitoring, and discussing, and the smaller grain-size core practices of probing, revoicing, explaining, pressing, and orienting, which we outlined as instructional moves that cut across a lesson and could be used to meet different goals related to each of the larger practices.

Finally, our fourth design principle was based on both our commitment to respect teachers as professionals with expertise and the research focus of the broader project. From the literature on rehearsals for prospective teachers, teacher educators often provide forms of feedback that are directive or evaluative (Lampert et al., 2013). For us, it was important to modify the purpose of pausing rehearsals to focus on publicizing the in-the-moment instructional decisions teachers were making during rehearsal. Thus, we chose to focus feedback on eliciting the goals for teachers' instructional moves or hypothesized future decisions to understand the ways teachers negotiated meaning of the practices.

We conjectured broadly that rehearsing core practices would support teachers in making meaning of the practices with a more ambitious purpose. That is, rehearsing a practice would problematize their existing practices and provide opportunities for imaginative practice of new or repurposed core practices or instructional moves that teachers could use to teach in ways that were more ambitious.

**Rehearsals in Professional Development**

In each summer institute, all teachers engaged in the three rehearsals by taking on the role of the teacher, student, or observer. For the launching and monitoring rehearsals, one participant served as teacher and rehearsed, while three participants simulated students based on the student profile cards. For the discussion rehearsals, pairs of teachers shared the role of teacher and selected, sequenced, and orchestrated a discussion around student approaches as the other teachers simulated students. During each rehearsal, a MTE served as a facilitator, periodically pausing the rehearsal to elicit the purpose for teachers' enactments of the practice, the goals for their instructional moves, or facilitating the conjecturing of future possibilities in the rehearsal. After each rehearsal, participants reflected on what they learned through small group discussions and written reflections.

To demonstrate how we engaged teachers in rehearsal, I offer one example of our cycle of investigation and rehearsal for the core practice of monitoring small groups from the first implementation. Following this, I present evidence of two teachers' engagement in rehearsals of the core practice of monitoring across their participation in two implementations of the summer institute portions of our PD.

**Cycle of investigation.** Our cycle of investigation began with participants engaging with a cognitively demanding mathematics task, while a MTE modeled ambitious teaching. Because the PD was for secondary mathematics teachers who taught several different courses, we chose tasks that allowed for multiple approaches and could be adapted to meet multiple standards of high school mathematics. After teachers experienced an ambitious lesson, we elicited from participants broad characteristics of the lesson and particular instructional moves that supported their learning. One of the MTEs facilitated the discussion and directed it to the focal core practice of the day, in this case monitoring student's engagement with the task. After discussing different teachers' routines when monitoring, we collectively codified this discussion into a framework for the practice that focused on the purpose of the practice and goals for moves that teachers could use to support students toward the purpose of the practice (Appendix A).

After we discussed the framework in relation to ambitious teaching, we engaged teachers in the approximation of practices shown in Figure 8. During this professional learning task, teachers watched a classroom video, identified instructional moves, inferred goals for their use, conjectured potential moves that could support meeting the purpose of the practice in response to the ways the teacher and students were engaging with the task, and summarized the overall purpose of monitoring for the teacher. This approximation aimed to support teachers in understanding the purpose of the practice and goals for moves they could use.

**Monitoring rehearsal.** The task we chose to design our rehearsals around could be approached by creating a table, exploring patterns, using the recursive form of a geometric sequence, drawing upon knowledge of exponential functions to engage with the explicit form of a geometric sequence, or graphically by developing a logistic model, among others (see Figure 11 for the task). It could be used to meet several important algebra and functions standards that cut across multiple secondary courses, including F.IF.4, 7,8; F.BF.2; F.LE.1; A.CED.1 (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

**Medicine Task**

*A student strained her knee in a cheerleading competition, and her doctor has prescribed an anti-inflammatory drug to reduce the swelling. The student takes two 220mg tablets every 8 hours for 10 days. Her kidneys eliminate 60 percent of this drug from her body every 8 hours. How much of the drug is in her system after 10 days? If she continued to take the drug longer than 10 days, how much of the drug would be in her system?*

Figure 11. Medicine Task (Adapted from NCTM, 2009).

To help teachers prepare to rehearse, we gave them time in groups to revisit the mathematical task, anticipate strategies and barriers they predicted students may have, and generate lists of instructional moves they could using in relation to their anticipations to achieve goals outlined in the framework. We then created groups of eight, which was convenient for our context and the time we had allotted to rehearse. For each rehearsal group, three teachers were assigned different student profiles (see Figure 10 for examples) and given time to understand the "student" they would be representing in the simulated small group. During this time, the rehearsing teacher, unaware of which student profiles the other teachers were taking on, prepared to rehearse by reviewing their anticipations and potential moves they could use during the rehearsal. The remaining four teachers acted as observers and focused on the rehearsing teachers' enactment of the practice, including instructional moves, corresponding goals, and the overarching purpose of monitoring.

Each rehearsing teachers was free to begin the rehearsal in any way they felt comfortable. Some teachers chose to act as if they were in the middle of the lesson by approaching the small group as if they had interacted with them previously, stating, "Last time I was here, I saw that you were making your new/old and total table right here (see Example #1, Figure 10). Can you explain to me what is going on for the next steps?" Others chose to act as if it was the first time they had engaged the group. After observing the students' written work, one rehearsing teacher who used this approach said, "Okay, I notice you two are working on the same thing, so how about we start with you guys and you guys tell me what you are doing." As each teacher rehearsed, the teachers acting as students drew upon the student profile cards to support them in responding to the rehearsing teacher's questions.

As the rehearsal progressed, a MTE paused the rehearsal periodically. Sometimes, these interactions were intended to uncover the goals for the rehearsing teachers' moves by broadly asking, "I'm curious about what you just did. Why did you do what you did?," or more specifically, "tell me about your questions, so you made it an explanation question and invited them to explain, then you just asked a question. Tell me what you are trying to do?" Other times, these interactions were to elicit instructional decisions the rehearsing teacher was considering making in response to students' mathematical thinking, such as, "you left him with, "see if you can find the pattern" - and I feel like you made sense of what they were doing, so what are you thinking about doing next?" A rehearsal concluded when the rehearsing teacher felt they had engaged productively with the small group and simulated leaving the group, or when the facilitator felt as if the rehearsing teacher had engaged in the rehearsal productively and it was time for another teacher to rehearse.

Each rehearsal typically lasted about five to ten minutes depending on the core practice being rehearsed and the ways teachers engaged in the rehearsal. At the conclusion of each rehearsal, teachers took time to first individually reflect in writing by answering questions about the meaning they were making of the practice, the goals from the framework they felt comfortable enacting or found more difficult, and how the purpose of the practice and the goals in the framework supported their overall vision of mathematics teaching. After this reflection, teachers had the opportunity to engage together in a short debriefing of their rehearsal to address any lingering questions or conjectures before participants switched roles for another rehearsal.

This extended example also illustrates how our first, second, and fourth principles shaped the rehearsal design. More specifically, the example shows how the rehearsal was grounded in a common task, enacted around profiles of students' mathematical thinking, and facilitated to expose the purpose and goals for teachers' broad enactment and their instructional moves. Our third principle concerned rehearsing sequential core practices to focus on enacting whole lessons – by rehearsing the practices of launching, monitoring, and discussing over the course of the summer institute, we were able to guide whole group discussions and teacher's engagement in rehearsals while remaining focused on the mathematics of a common task, and maintained a broader attention to a learning goal for a whole lesson.

## Considering Relationships Between Enactments and Rehearsals

The two implementations of our PD took place during 2015-2016 and 2016-2017 in one midsized, rural school district in the southeastern United States. Across the two implementations of the PD, 19 mathematics teachers who taught high school mathematics courses volunteered to participate, with eight of these teachers participating in both years of the PD. In this paper, I present evidence of two of these teachers' participation in monitoring rehearsals in the summer institutes, briefly relating their rehearsals to their classroom enactments of monitoring across five lessons they taught over two years.

Elsewhere, I reported on the ways these two teachers, Dawn and Sara, enacted the core practices of launching, monitoring, and discussing in their classroom teaching

(Webb, in preparation b). To analyze the ways these teachers' participation in rehearsals might relate to changes in their enactments of core practices, I used a constant comparative approach (Glaser, 1965; Strauss & Corbin, 1998) to relate teachers' participation in rehearsals to changes in their enactments. Data for these analyses consisted of summaries of each teachers' five lessons and transcripts of teachers' rehearsals and post-rehearsal debriefings, their written reflections, and transcripts from focus group interviews conducted twice during each implementation of the summer institutes.

In the following section, I briefly summarize changes in these two teacher's enactments of the practice of monitoring and then share examples from their monitoring rehearsals in both summer institutes. I begin by sharing summary quantitative measures of each teacher's enactments of monitoring, reported in greater detail elsewhere (Webb, in preparation b). I present measures (on a scale from 0 to 4) that represent the degree to which teacher's met the purpose of the practice promoted in the professional development (*Purpose)*, the degree to which they pressed students for conceptual explanations or to extend their mathematical thinking (*Pressing*), and the degree to which teachers connected students' contributions and showed how these contributions related to each other (*Orienting*). The instructional measures were based on modified versions of a subset of rubrics used by the project team (Junker, et al., 2005) and summarized elsewhere (Webb, in preparation, b). I then share brief qualitative summaries of each teachers' enactments and share examples from their rehearsals as evidence of the ways the design features of the rehearsals and teachers' engagement in rehearsals may have supported their enactments of each core practices. The summaries of teachers' enactments are not meant to be exhaustive, but rather to serve as context for understanding relationships to their engagement in rehearsals.

**Monitoring Students Engagement in Mathematics**

As shown in the Questioning column in Table 10, Dawn's and Sara's enactments of the practice of monitoring advanced toward the purpose of monitoring promoted in the professional development, which was to support students in engaging with the task in ways that advance toward a mathematical goal (see Appendix A for monitoring framework). While Dawn's enactments of monitoring improved and Sara's enactments of monitoring maintained the proposed purpose, their use of pressing (Kazemi & Stipek, 2001) and orienting (Boaler & Brodie, 2004; Boaler & Staples, 2008) moves to support students evolved across their enactments, and in some ways, differed in both quality and intent.

Table 10

Monitoring Enactment Scores for Dawn and Sara

| **Instructional Quality Scores [0 - 4]** | | | | | | |
|---|---|---|---|---|---|---|
| **Lesson** | **Dawn** | | | **Sara** | | |
| | Questioning | Pressing | Orienting | Questioning | Pressing | Orienting |
| **1** | 2 | 2 | 0 | 4 | 2 | 0 |
| **2** | 4 | 3 | 1 | 3 | 3 | 2 |
| **3** | 4 | 4 | 2 | 4 | 3 | 2 |
| **4** | 4 | 4 | 4 | 4 | 4 | 2 |
| **5** | 4 | 3 | 4 | 4 | 4 | 2 |

**Dawn's monitoring enactments and rehearsals.** For Dawn, her purpose for monitoring evolved from leaving students to explore on their own to supporting students to persist with their mathematical conjectures and relating students' strategies together. She used pressing moves throughout her enactments and over time incorporated more orienting moves, the quality of which progressed to be more responsive to students' thinking, as seen in her fourth and fifth enactments in Table 11.

During Dawn's first rehearsal in the first summer institute she predominantly used probing moves to uncover students' thinking and pressing moves to encourage students to think more deeply about the mathematics of the task. In her post-rehearsal reflection, she noted her need to consider using pressing moves in instruction stating, "I am better at probing than pressing...[I wonder if] this is due to my level of preparedness or lack of practice."

In Dawn's second rehearsal in the following year's summer institute, she used probing, pressing, and orienting moves to ensure students were making progress toward her learning goal. At the conclusion of an exchange with students related to their mathematical representations, she had the following exchange with the MTE:

> Dawn: What I am noticing is that S1, your picture and S2's picture looks a lot alike. But S2 seems to think it is linear and you are thinking it is quadratic. At this point, there needs to be some conversation with you guys about where to go from here. If you think you are right, you need to convince S1. Same for you S2. When I come back I want to be able to move that a little bit further along.
>
> MTE: So, tell me about that one.
>
> Dawn: That takes me out of the picture, gives them some meaningful conversation, and gives me the opportunity to check in with S3 and pull her in as well, and then the opportunity to move along [to another group]....he is using standard form of the equation, so I feel like this table, if my goals is to get to the standard form of the equation – this group is going to be able to take me there. I can hopefully find another group that is working on a different format and be able to make connections back.

In this exchange, Dawn indicates that while rehearsing monitoring she is making meaning of managing the complexity of multiple mathematical ideas within a group and across multiple groups to ensure students are making progress in meeting her learning goal for the lesson.

**Sara’s monitoring enactments and rehearsals.** For Sara, she began her participation in the professional development with a purpose for monitoring of supporting students as they used their own ideas to engage with the task, predominately using probing and explaining moves during her first enactment. Over the course of her two years of participation, her purpose of monitoring changed incrementally and at the conclusion of her two years of participation was best described as supporting students to engage more deeply with the mathematics of the task and ensuring that students engaged with each other’s mathematical thinking. This shift in purpose was accompanied by an increased use of pressing moves and goals for pressing and orienting moves that were more focused on responding to student’s mathematical thinking as indicated in Table 11.

**Sara’s monitoring rehearsals.** In her monitoring rehearsal in the first summer institute. Sara predominantly used probing moves to understand students’ thinking, and after engaging with one student in the group for an extended period of time, she had the following conversation with a MTE.

> MTE: Pause for a second. So, these are your last four questions, or actually the four questions you’ve asked: “Where are you? What are we trying to find? What was our question? Have you found it yet?” Can you tell me what you are trying, what is S1 giving you and what are you still trying to get?
> Sara: So, I’m trying to assess where she’s at and if she’s finished in her mind or if she’s going to go forward.
> MTE: So where are you going to go next?

> Sara: I've been debating should I move away from her and go to other students, try to make connections here, or go to the other students – I don't know.
> MTE: So, what would going to the other students [S2 & S3] do?
> Sara: Well, if I heard what S2 or S3 said and then said, "see what you all can come up with", I would probably leave it as a group, now that I know where they are at...I would probably just leave them for a little while and go to another group.
> MTE: Okay, what would it not give you? What if you stuck with S1?
> Sara: I wouldn't allow them time to communicate.
> MTE: Decisions, decisions, right?

In addition to conjecturing about supporting a single student versus making sense of what other students in the group were doing, Sara noted in her reflection that:

> choosing questions that do not "push" students to a certain approach, but "pulls" them to start thinking and make mathematical connections is a struggle for me. This is not the type of questions I have used in my classroom often. It is easier for me to probe because I use these characteristics in my classroom already.

During Sara's second monitoring rehearsal the following summer, the MTE paused her rehearsal to ask about what she knew about the different approaches of each student. After this exchange Sara used several orienting to ensure that students were working together toward her mathematical goal by asking questions such as, "S1, I see that you have the same table as S2 except you didn't put this pattern. Can you revoice what she said about the pattern?" She concluded her rehearsal by using a pressing move, stating, "What I am hearing from everybody is that as a group you have a lot of knowledge. I am going to let your group work on answering the question you just asked – does it have to be linear or can it be quadratic."

In Sara's post-rehearsal debriefing she noted the difference in the meaning she was making of the core practice of monitoring, saying:

> In the past, I would just monitor to check for understanding or what they know, but I never had the drive to get to a certain goal in terms of more understanding or new content. So, to me by practicing [rehearsing] I am constantly thinking about that I am not just assessing what they know and what they currently understand, but where are we going with this. In this setting you are not looking for everybody to have the same thing...and not looking for everybody get the answer, but the understanding they need to have to reach the goal.

Evidence suggests that Dawn and Sara's participation in two cycles of rehearsals across the two summer institutes related to changes in their enactments of the core practice of monitoring. For Dawn, her first rehearsal provided the space to refine her purpose for the practice and try out and reflect on her use of pressing moves. In her second rehearsal, she was able further refine the purpose of the practice and relate the rehearsal with one small group to the imaginative work of managing the complexity of whole class instruction toward a mathematical goal.

In Sara's first rehearsal she problematized the typical moves she used during monitoring and made new meaning of moves she could use that were more responsive to students. In her second rehearsal, she continued to find it generative and tested orienting moves with goals that aligned with the purpose of monitoring promoted in the professional development and toward the mathematical goal of the task.

## Discussion

We designed rehearsals as a space where secondary mathematics teachers could engage in imaginative enactments of core practices that leveraged and built upon students' mathematical thinking. The examples I shared indicate that rehearsals provide opportunities for teachers to take risks and explore alternative ways of doing things, and that the meanings teachers made in rehearsals might relate to changes in their enactments of these practices in their classroom.

Our rehearsals included several design principles important in supporting teachers in their participation in rehearsals. First, rehearsals grounded in a common task support teachers by reducing the complexity of the mathematics so teachers can focus on the purpose of the practice and make decisions about moves they can use to support students. Second, designing student profile cards that outlined a barrier to engaging in the task, a mathematical approach, and a sample of student work support teacher's engagement in rehearsals, for both the rehearsing teacher and teachers playing students. In each example I shared, teachers used these profiles to act as students in ways that supported the rehearsing teachers' engagement in the core practice.

Third, the feedback provided by MTEs was not evaluative. Rather, the feedback respected teachers' expertise, their own goals for improving their practice, and the complexities and multiple instructional decisions teachers make when teaching. Focusing on feedback that elicits the purpose or goals for teachers' decisions promotes reflective practice, positions teachers as generative learners, and fosters a norm of publicizing practice for collective learning (Lieberman & Pointer-Mace, 2009). Finally, designing rehearsals to successively build across a lesson supports teachers in gathering images of each core practice in ways that attended to the dynamic and contingent aspects of teaching a whole lesson.

In this paper, I provided evidence of the ways teachers' classroom enactments related to their participation in each cycle of rehearsals. This evidence is not meant to support a causal claim, but rather it indicates that, as a part of a practice-based PD, teachers find rehearsals to be both productive and generative – offering continued opportunities for learning to learn from practice over multiple years of rehearsals. After their participation in the first implementation, we were skeptical of whether teachers would find rehearsals useful in the second implementation. To our surprise, their comments continually suggested otherwise. In Figure 12, I provide two excerpts from both Dawn and Sara's comments during focus groups highlighting the ways they found rehearsals useful across the two summer institutes.

| | Dawn | Sara |
|---|---|---|
| Summer Institute #1 | This[rehearsing] has given us a chance to move from just being a witness of the modeling to actually taking those baby steps of how to do this a better way in our classrooms, giving us some hands-on opportunity. | I felt like that it [rehearsing] just started me thinking about how much I need to think about as a teacher that I've never thought about before. |
| Summer Institute #2 | This is invaluable to do this. I think sometimes teachers are hesitant to change, they know they need to change and try something different, but because they don't have the opportunity to practice and explore what that looks like and feels like, they just don't do it. This is exactly what I think every teacher really needs, the raw experience of doing it without the fear of failure. | The first year you are so involved in trying to understand what everything is. What is a discussion, what is a launch, and how are they supposed to look? This year I have been able to relate it more...and now that I have had time to process it...I know there is a learning goal, [and getting] to that learning goal is what is clearer to me now that was last year. |

Figure 12. Teachers' Comments Related to Rehearsing Core Practices.

As we consider future implementations of practice-based PD that include rehearsals, we make several recommendations for MTEs interested in similar work. First, our efforts to redesign the PD for the second implementation was informed by requests from teachers to add two additional rehearsals to support them in teaching whole lessons. First, they wanted to add a "closure" rehearsal to practice the work teachers do after a whole class discussion to formalize the mathematics developed throughout the lesson and move toward providing students opportunities to engage in additional problems related to the mathematical goal.

Second, teachers wanted to add a "warm-up" rehearsal to practice multiple core practices at once, similar to the instructional activities that are often used with prospective teachers. In our second implementation, we added a closure and warmup rehearsal for returning teachers. However, we have yet to explore whether their desire for additional rehearsals was due to their increased understanding of ambitious teaching gained from their participation in the first iteration of the PD, or if this would be a productive approach to the overall initial design. This is a design consideration others should weigh, however, the fact that teachers asked for additional rehearsals further suggests that they find them useful in trying out core practices over multiple years as they continue to work on their teaching.

Another design consideration extends our commitment to valuing the expertise of teachers who seek opportunities to learn and improve by attending professional development. For Dawn and Sara, rehearsing each of the core practices with the frameworks was productive in both implementations of the summer institute. However, when analyzing other teachers' classroom enactments, their engagement in rehearsals did not relate as closely to the changes in their enactments or their enactments did not progress forward toward more ambitious purposes. In future implementations, we would like to consider providing teachers more agency in choosing their own pedagogical goals that they would like to work on in rehearsal. We conjecture that for some teachers who already enact these practices in their classroom, increased agency would provide them a space to engage in rehearsals with greater attention to issues of students' mathematical identity, equity, or social justice in mathematics.

Finally, I conclude by providing two notes of caution in light of these conjectures for future designs of rehearsals. First, we spent several days building and establishing a participation structure that fostered a learning environment conducive for teacher learning in rehearsal prior to using rehearsals. To do so, we were intentional and deliberate to make our own practice public (Lieberman & Pointer-Mace, 2009) for analysis and critique. Each day, we providing teachers with extended opportunities to ask questions and push hard on our own pedagogical practices and humbly noting areas where we could improve our practice as MTEs. This authenticity and honesty was imperative for creating a caring space where teachers felt comfortable rehearsing with us and their colleagues (Sztajn, 2008; Sztajn, Hackenberg, White, & Allexsaht-Snider, 2007). Thus, we advise against attempting to use rehearsals as a stand-alone MTE pedagogy. Second, while we found the use of the student profile cards central to supporting enactments of the rehearsals, we conjecture that adding other features to the profiles that provides more information about the "hypothetical students" may evoke existing narratives about students that are potentially deficit-focused (Wilson, Sztajn, Edgington, Webb, & Myers, 2017). We see this conjecture as a potential site for future design-research efforts.

This paper addressed calls for practice-based learning opportunities for practicing teachers (Anthony et al., 2015; Garet, Porter, Desimone, Birman, & Yoon, 2001, Grossman & McDonald, 2008; McDonald, Kazemi, & Kavanagh, 2013). To do so, we built from the existing literature on rehearsals for prospective teachers to design rehearsals to support teacher learning in PD. Rehearsals for teachers proved to be a productive space for MTEs to support teacher learning of practice and we see these efforts as future sites for continued research on the possible uses of rehearsals and the threshold of what they can accomplish. We encourage others interested in practice-based PD to build from and improve these efforts

# CHAPTER V

# CONCLUSION

In this dissertation, I set out to examine a conceptual model of teaching to support design and research efforts of mathematics teacher learning of practice in professional development. In the introduction, I laid out the motivation for a study focused on teacher learning of core practices by describing reform goals for the mathematical proficiency of students (National Research Council, 2001; CCSSM, 2010) and a forward-looking vision of what ambitious teaching could look like to meet these goals (Kazemi et al., 2009). Next, I highlighted the work of researchers' and teacher educators' to identify core practices of ambitious teaching and the potential of designing learning opportunities for teachers to deliberately try out practices in rehearsals (Ball & Cohen, 1999; Grossman & McDonald, 2008; McDonald, Kazemi, & Kavanagh, 2013, Sandoval et al., 2016). I concluded by highlighting challenges to this approach, the lack of conceptual and empirical literature attending to teacher learning of core practices, and the potential that a shared conceptual model might have in making progress in accumulating knowledge and building theory of teacher learning of practice(Clarke & Hollingsworth, 2002; Opfer & Pedder, 2011).

## Review of the Three Manuscripts

With this dissertation, I investigated three questions regarding the conceptualization, design, and analysis of teacher learning of ambitious teaching:

1) How can teaching be conceptualized to inform research and design for teacher learning that both respects and challenges teachers' existing practices?,
2) In what ways do teachers recompose cores practices together across their participation in two years of professional development focused on practices of ambitious teaching?, and
3) What is a design for rehearsals in professional development that supports teachers in learning core practices of ambitious teaching?

The three manuscripts that comprise this dissertation contributed to each of these questions and, in general, help develop a more robust and informed notion for research on, and the design of, pedagogies that can support teacher learning of ambitious teaching. Separately, they offer insight into different aspects of the larger work which I summarize below.

**Manuscript 1**

The first manuscript addressed the first question by exploring the ways teaching could be conceptualized using a core practice approach to both respect and challenge teachers existing conceptions of teaching. I considered a set of design considerations (Jacobs & Spangler, 2017) and learning tensions (Jansen, Grossman, & Westbroek, 2015) of a core practice approach, and investigated hierarchical modularity (Simon, 1996) as a way to organize practice to reconcile these challenges. I did so with explicit attention to the work mathematics teachers do as they navigate the complexities of relating learning in professional development and their evolving conceptions of practice. I drew upon data and analyses from across teachers' participation in rehearsals and enactments of core practices in their teaching to provide examples to support my theoretical analysis. From these data, I illustrated the ways that hierarchical modularity is a useful way to conceptualize practice for both design and research.

For design, I showed how hierarchical modularity can support efforts to design professional development focused on practice, organize core practices of various grain sizes, and attend to the multiple aims for enacting ambitious teaching. For research, I showed how hierarchical modularity is useful for investigating the ways teachers: recompose core practices to enact whole lessons; refine both the skills of teaching and their aims for enacting practices; and adapt existing routines in response to students' thinking about content and their existing conceptions of practice.

**Manuscript 2**

The second manuscript detailed a retrospective analysis of the ways four teachers brought together responsive instructional moves to recompose the core practices of launching, monitoring, and discussing. Bringing together hierarchical modularity (Simon, 1973) and Wenger's (1998) notion of boundary encounters, allowed for an in-depth understanding of the ways teachers recomposed their practice at different levels of the system. My attention to nested core practices of varying grain-sizes and the purposes and goals for enacting these practices provided empirical evidence of the complexity of the relation within and across practices of teaching and their effects on the quality of instruction.

From these analyses came two sets of findings. First, results indicated that for all teachers, their use of pressing and orienting moves propagated to produce larger changes within each core practice that can be seen as more ambitious and responsive to students' mathematical thinking. Second, results also indicated that teachers recomposed their lessons to include the three large-grain size core practices in different ways, and that launching, monitoring, and discussing could be seen as increasingly complex core practices over the course of a lesson. These findings strengthen the argument that changes in smaller practices can support improvements in larger practices and that using hierarchical modularity is a productive way to both design for and study teacher learning of practice.

**Manuscript 3**

Across the first and second manuscripts, I primarily drew upon data from teachers' lessons. In the third manuscript, I described and drew upon data from teachers' engagement in rehearsals of core practices and used findings from the second manuscript to motivate an exploration of the relationship between changes in teachers' enactments of core practices and their engagement in rehearsals. I described rehearsals designed for use in professional development with secondary mathematics teachers by detailing our design process, presenting ways in which teachers engaged in rehearsals in professional development, and providing evidence of the ways two teachers' engagement in rehearsals supported them in imagining new ways of teaching that aligned with changes in their classroom practice. I concluded this manuscript with several design considerations, revisions to our rehearsals, and a discussion of the role of mathematics teacher educators in supporting teacher learning of practices of ambitious teaching. This paper provides teacher educators both a concrete example to incorporate into their own practice, as well as the potential of rehearsal as a future site for research for teacher learning of practice.

**Crosscutting Findings**

Collectively, the three manuscripts represent a conceptualization of practice and retrospective analysis of changes in teachers' classroom practices and ways these changes relate to teachers' engagement in rehearsals in professional development. Across this study, I would like to highlight two findings that build from and extend the existing literature.

First, findings from the empirical manuscript extend the findings of others highlighting the benefits of teachers' use of responsive instructional moves. Related to pressing moves, Kazemi & Stipek's (2001) research emphasizes that exchanges with students that press them to go beyond superficial descriptions of their mathematics to justify their reasoning or consider alternative strategies, benefits student learning. My findings extend this research to suggest that not only do pressing moves support student learning, they also support teachers in achieving their learning goal for a lesson. Similarly, my findings also extend research emphasizing the implications of orienting students to one another's mathematical thinking (Boaler and Brodie, 2004; Boaler & Staples, 2008). These researchers found that orienting moves benefit student learning, but that these moves are often not a part of teachers' existing practice. Findings from the empirical manuscript corroborate that teachers' practice prior to their participation in a practice-based professional development did not typically draw upon orienting moves. However, as they progressed throughout their participation they grew to use these moves within and across different practices and their orienting moves improved their enactments of the larger practices.

Second, these findings support my initial conjecture that small changes in lower level instructional practices that are more responsive to students' thinking can support more ambitious teaching. To develop these findings, I drew upon hierarchical modularity (Simon, 1973) to conceptualize practice in a way that allowed me to attend to both these small changes in practice and the ways they impacted multiple larger practices. Drawing on a supporting theory of learning (Wenger, 1998) this dissertation extends the fields current conceptualizations of practice. In doing so, it answers the call to design efforts for professional learning to focus on the practices of teaching and a complementary reconceiving of how we bring together conceptual tools and theories to research teacher learning (Kazemi & Hubbard, 2008; Clarke & Hollingsworth, 2002; Opfer & Pedder, 2011). In doing so, this model might further the field in making progress in accumulating knowledge and building theory of teacher learning of practice.

## Limitations of This Study

As with any study, the conceptual, empirical, and design efforts described throughout this dissertation has a number of limitations. Core practices research and design are relatively new and developing. In this dissertation, I sought a conceptual and theoretical framework to attend to complexities of a core practice approach, thus, these limitations are not so much shortcomings as they are boundaries set around the claims that can be made from these particular data and analyses, and the theoretical perspectives

I chose to draw upon. In this section, I highlight limitations from each of the three manuscripts.

**Manuscript 1**

In this conceptual paper, I considered hierarchical modularity as a way to reconcile the challenges of a core practice approach and provided an existence proof that hierarchical modularity could be a useful way to conceptualize practice for both design and research. While I noted several useful outcomes of these analyses, I also noted several limitations and considerations for future research. First, a sizeable premise to a hierarchical modular approach is the need to attend to externalized action and make inferences about aims. Because this theory has traditionally been used in physical systems, I noted that from a social science perspective, inference of the aims for teachers use of core practices of different grain sizes is necessarily difficult.

Second, because hierarchical modularity focuses on externalized action, it fails to attend to less visible aspects of teaching such as the ways teachers position students as learners (Wilson, Sztajn, Edgington, Webb, & Myers, 2017) or teacher noticing (Jacobs, Lamb, Philipp, 2010). It also does not attend to the ways in which teachers might delay decision making during instruction for unseen reasons. That is, my choice to infer aims during instruction requires attention to the temporal or in-the-moment aims while failing to attend to the possibilities of teachers delaying a decision for later in a lesson and the fact that teachers manage multiple goals simultaneously during instruction. Also, these goals can often be in conflict and are not always related specifically to teaching content. For example, teachers are always managing content goals, affective or social goals, justice or equity-based goals, and others. My attention to mathematics teaching from a specific set of core practices and grain sizes fails to take these goals into account. Further research could explore adding these goals to a hierarchical modular approach.

**Manuscript 2**

Highlighting the limitations noted in the first manuscript, in the second manuscript I brought together boundary encounters (Wenger, 1998) as a situated theory of learning to address the ways teachers bring elements of their existing practice to the professional development and over time incorporate elements of the professional development back in their classroom practice. In this paper, I drew upon the characteristics of internal and vertical coupling for an in-depth analysis of the ways teachers recomposed core practices at different levels of the system. In addition to the limitations described for the first manuscript, using the model for research surfaced several additional limitations important to consider. First, I noted the complexity of mathematics teaching and the fact that there is much more to the work of mathematics teaching than a subset of core practices of varied grain sizes. By focusing on three large-grain size core practices and two instructional moves, I suspended attention to other core practices of varied grain size. In addition, this study suspended attention to both non-interactive core practices such as designing lessons or setting learning goals, and interactive practices such as classroom management, fostering student agency, attending to issues of race, or implementing norms that are not necessarily subject specific.

Second, this study analyzed only five lessons over multiple academic years and across two implementations of a professional development. Thus, teachers' enactments in these lessons should not be seen as representative of teachers' daily practice, and changes in practice should be recognized within the context of this timeframe and their commitment to enacting these lessons as part of their participation in the professional development. While this study has these limitations, it also motivates the need for further studies that attend to shorter timespans between lessons to strengthen the research base on teacher learning of practice across settings.

**Manuscript 3**

This paper addressed calls for practice-focused learning opportunities for teachers (Anthony, Hunter, Hunter, Rawlins, Averill, Drake, et al., 2015; Garet, Porter, Desimone, Birman, & Yoon, 2001, Grossman & McDonald, 2008; McDonald, Kazemi, & Kavanagh, 2013) and built from the existing literature on rehearsals purposed for prospective teachers (e.g. Lampert et al., 2013) to design rehearsals to support teacher learning of practice in professional development. Here, rather than focusing on limitation, I highlight two important considerations to designing rehearsals for teachers.

First, the work teacher educators must to do to build a community of care, vulnerability, and authenticity around the complexities of mathematics teaching is imperative for providing teachers a space to feel comfortable rehearsing core practices (Sztajn, 2008; Sztajn, Hackenberg, White, & Allexsaht-Snider, 2007). Thus, attempting to use rehearsals as a stand-alone pedagogy is not advised. Second, while the use of "student profile cards" brought into focus students' mathematical thinking and supported teachers engagement in rehearsals, I cautioned that adding other features to the profiles that seek to further explicate "hypothetical students" may serve to reify existing narratives about students that are potentially deficit focused (Wilson et al., 2017). Third, the design efforts highlighted in this dissertation is one of few considering rehearsals designed for teachers, thus further research needs to explore the affordances and constraints of such an approach and whether other theoretical perspectives can bring into focus different design challenges.

## Implications for Practice, Research, and Policy

All of these limitations and the need for future design-research studies are the product of innovative work that is just beginning. Even with these limitations, my efforts in this dissertation highlight progress made in conceptualizing practice for designing practice-focused and responsive pedagogies of teacher education. Thus, these limitations motivate continued design-based research in the field of mathematics teacher education. I conclude by briefly outlining implications for practice, research, and policy.

### Implications for Practice

Teacher educators have the dual goals of supporting teacher learning of practice, and in doing so, also supporting student learning in classrooms. Findings from this study suggest three important issues related to improving teaching. First, because the core practices and instructional moves explored in this dissertation have been shown to have impacts on student learning, changes observed in teachers' practice toward more ambitious forms of teaching will likely benefit student learning. Thus, both the pedagogy of rehearsal and conceptual model of practice introduced and explored in this study warrants attention for its potential. Second, and relatedly, while the hierarchical conceptualization of practice I shared in this study was mainly for design and research efforts, this conceptualization provided a common way to talk about practice with teachers. Consequently, I see this model as being useful for multiple stakeholders to discuss and examine practice in both school-based and university-based settings. Moreover, it may benefit efforts aimed at bringing together prospective and practicing teachers for learning of practice. Third, outcomes from the empirical manuscript highlight that even small changes in teachers' use of responsive instructional moves, can have profound impacts on teachers' enactments of larger practices – dually benefiting students' learning and teachers' practice. For teachers, who are constantly managing the challenges of shifting educational policies and expectations of their practice, the idea of making small changes to practice might resonate with teachers who desire to improve student learning outcomes.

**Implications for Research**

As teacher education researchers, our research is intricately tied to our designs for teacher learning. More specifically, as design-researchers, to further advance our understanding of teacher learning, we must explicate the design principals and learning conjectures that are embodied in our design. While these principles and conjectures are useful for design, for researchers interested in also building theory, greater attention to the ways principles and conjectures impact learning and are modified over time support the development of learning theory. As Sandoval (2004) states,

> the systematic study of designed interventions can develop learning theory because designed learning environments embody design conjectures about how to support learning in a specific context that are themselves based on theoretical conjectures of how learning occurs in particular domains (p. 215).

Throughout this dissertation, I highlighted conjectures related to teacher learning of practice, and in the practitioner manuscript, explicitly laid out my design principles. Future research efforts focused on teacher learning of rehearsals, both in prospective and practicing teacher education, would be well served by greater attention to the development of conjectures and principles for design if we are to build theory related to teacher learning of practice.

In this study, I brought together hierarchical modularity (Simon, 1973) with a situated theory of learning (Wenger, 1998) to conceptualize practice and investigate teacher learning across settings. The conceptual work discussed in the first manuscript made meaningful progress toward furthering the fields efforts to explore teacher learning using a core practice approach. The empirical analysis in the second manuscript provided new ways learning can be explored across teachers attempts to enact practices in their classroom teaching. The practitioner piece offered implications for design and research of teacher learning in professional development. Bringing together these two frames warrants consideration from the field regarding its utility in attending to teacher learning of practice. Further research could explore the ways different theories of learning impact both the conceptualization I put forth and the limitations because of it.

**Implications for Policy**

As teacher educators and researchers, we all play a role in the development, interpretation, and enactment of policies that relate to teaching and learning. As a field, we continue to build a case for what constitutes effective learning opportunities for teachers, namely, that they be intensive and ongoing; connected to content, practice, and students' thinking; encourage shared participation; and be built with attention to adult learning theories (Darling-Hammond et al. 2009; Desimone, 2009; Elmore 2002; Heck et al. 2008; Sztajn et al., 2007; Yoon et al. 2007). My choice to investigate the practices of teachers who participated over two years of our professional development further highlights the benefits of longitudinal learning of practice and the generative opportunities it provides for teachers. In our role as policy influencers, this provides further evidence for longitudinal professional development focused on practice that we can use as we interact with various education stakeholders.

## Concluding Remarks

I see the dissertation presented here as both a productive contribution to the field of teacher education broadly, and mathematics teacher education specifically. The theoretical and empirical contributions of this dissertation further the fields' calls for attention to learning of practice, bring into focus an organized way to manage and research ambitious teaching, and highlight the possibilities for accumulating knowledge and building theory of teacher learning of practice. The design contributions provide an example of ways in which practice can be explored and worked on in teacher learning. While time will tell whether these ideas will be taken up by the field, I have found these contributions a productive space to think and learn, and will take them with me as I continue my research and my commitment to supporting teachers in developing into the practitioners they aim to be to support student learning.

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# APPENDIX A

## FRAMEWORKS FOR CORE PRACTICES

| **Launching** (e.g., Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013) |
|---|
| ***Purpose**:* Ensure students understand the mathematical goal of the task and can get started on the task. |
| • Allow students time to think about how they will approach the task.<br>• Ensure that students understand the context of the problem<br>• Address basic barriers regarding language and definitions as they relate to the mathematical ideas of the task or mathematical skills students may need to engage with the task<br>• Allow students to share approaches so all students have a chance to consider ideas or strategies<br>• Ensure that at least one member of each group knows how to get started |

| **Monitoring** (e.g., Stein, Engle, Smith, & Hughes, 2008) |
|---|
| ***Purpose**:* Support all students' in working on and engaging with the task in ways that advance toward the mathematical goal. |
| • Discover what students are thinking<br>• Draw upon your anticipations to understand students' approaches<br>• Encourage students who are not participating to engage in the task<br>• Orient students to one another's approaches<br>• Get students back on track if they are using an unproductive or incorrect approach, or an approach that does not support your mathematical goals for the lesson<br>• Encourage students who are taking a procedural approach to engage in thinking more deeply about a mathematical idea or why something does or does not work<br>• Identify approaches that will advance the goal of your lesson and can be used during whole class discussion |

| **Discussing** (e.g. Chapin, O'Connor, & Anderson, 2009) |
|---|
| ***Purpose**:* Facilitate discussion of students' mathematical work in relation to the mathematical goal. |
| • Select students to share whose ideas:<br>▪ lead toward the mathematical goal of the lesson<br>▪ represent common misunderstandings shared by students<br>▪ represent unique insight productive for other students to know<br>• Sequence the sharing of ideas in ways that move the discussion toward the goal by:<br>▪ building from less to more sophisticated ways of understanding the mathematical goal<br>▪ showing similarities and differences among approaches and connecting ideas<br>▪ enabling comparisons and contrasts of representations to highlight the mathematics of the lesson<br>• Connect across students' mathematical approaches to:<br>▪ assist students in making connections and reasoning about relationships between their ideas and others'<br>▪ prepare the groundwork for formalizing the mathematical goal |

## APPENDIX B

## ZIPLINE TASK

A new amusement park is building a zip line attraction. The attraction will have two towers on opposite sides of a man-made lagoon full of alligators. The lagoon will be 600 m wide. One tower will be 100 m tall and the other will be 60 m tall. There will be two zip lines, one from each tower, that riders will take from the tops of the towers to an island in the lagoon. Once on the island, riders will exit the ride by walking across a long bridge. But zip line wire is expensive! How far from the bank of the lagoon should the island be in order to minimize the length of zip line wire?

# APPENDIX C

# MODIFIED INSTRUCTIONAL QUALITY ASSESSMENT RUBRICS

**AR-Q: Questioning**

| | |
|---|---|
| **4** | The teacher consistently **(>3 for Launching, > 5 for Monitoring, Discussing**) uses academically relevant moves (**probing, pressing, orienting, revoicing**) that provide opportunities for students to elaborate and explain their reasoning, identify and describe important mathematical ideas in the task, or make connections between ideas, representations, or strategies; **AND 2 or more unique moves & goals from the frameworks are used/addressed** (i.e. orienting and probing, probing student thinking and orienting other to a students' approach) |
| **3** | At least 3 times (**during launching, monitoring, or discussing**), the teacher asks academically relevant questions (probing, pressing, orienting, revoicing); **AND 2 or more unique moves & goals from the frameworks are used/addressed** (i.e. orienting and probing, probing student thinking and orienting other to a students' approach) |
| **2** | • Less than 3 times (**during launching, monitoring, or discussing**), the teacher asks academically relevant questions (probing, pressing, orienting), OR<br>• **Uses only one type of move**, OR<br>• **Only meets one goal from the framework**, OR<br>• Teachers' moves that are superficial, trivial, or formulaic efforts to ask academically relevant questions (i.e. every student is asked the same question or set of questions) |
| **1** | The teacher asks procedural or factual questions that elicit mathematical facts or procedures or require brief, single word responses. |
| **0** | The teacher did not ask questions during monitoring, or the teacher's questions were not relevant to the mathematics of the lesson. |

**Note:** The goal is to characterize the degree to which teachers are teachers' moves are consistently attentive to students' mathematical thinking, are varied in the type of moves used, and meet several goals from the frameworks.

**AT-2: Teacher's Orienting**

| | |
|---|---|
| **4** | The teacher consistently **(>3 for Launching,  > 5 for Monitoring, Discussing)** connects/orients students' contributions to each other and provides opportunities for students to make connections by asking questions about how ideas/positions shared relate to each other, |
| **3** | At least 3 times **during launching, monitoring, or discussing** the teacher connects speakers' contributions to each other and provides opportunities for students to make connections by asking questions about how ideas/positions shared relate to each other, |
| **2** | There **are less than 3 times** during **launching, monitoring, or discussing** the teacher connects students' contributions to each other, but does not provide opportunities for students to connect how ideas/positions relate to each other OR no follow-up questions are asked after speakers' contributions. |
| **1** | The teacher revoices or recaps in ways that orient students' contributions only, but does not attend to how ideas/positions relate to each other |
| **0** | The teacher did not ask connecting or orienting questions during the lesson, or the teacher's questions were not relevant to the mathematics of the lesson. |
| **N/A** | Teacher did not enact the core practice |

Note: The goal is to focus on teachers' attempts to connect a students' mathematical work or ideas to the work or ideas of others in the group OR others in the class AND the ways in which teachers ask students to reason across these ideas or work.

**AT-4: Teachers' Press**

| | |
|---|---|
| **4** | The teacher consistently **(>3 for Launching,  > 5 for Monitoring, Discussing**) asks students to provide evidence for their contributions beyond simply sharing what they did (i.e. press for conceptual explanations, how do you know, tell me more about…), to explain/justify their reasoning, or **to extend their thinking to a new idea.** |
| **3** | **At least 3 times during launching, monitoring, or discussing** , the teacher asks students to provide evidence for their contributions beyond simply sharing what they did (i.e. press for conceptual explanations, how do you know, tell me more about…), to explain/justify their reasoning, or **to extend their thinking to a new idea.** |
| **2** | There **are less than 3 times** during **launching, monitoring, or discussing** where the teacher asks students to provide evidence for their contributions beyond simply sharing what they did (i.e. press for conceptual explanations, how do you know, tell me more about…), to explain/justify their reasoning, or **to extend their thinking to a new idea.** |
| **1** | **Most of the press is for computational or procedural explanations or memorized knowledge (their purpose is about facts, memorization, etc.)** |
| **0** | The teacher did not ask pressing questions during the monitoring, or the teacher's questions were not relevant to the mathematics of the lesson. |
| **N/A** | Teacher did not enact the core practice |

Note: The goal is to focus on teachers' attempts to press students to justify or explain their reasoning beyond their initial mathematical explanations or to press them to extend their thinking to a new idea.

# APPENDIX D

## CODEBOOK FOR INSTRUCTIONAL MOVES

| | | |
|---|---|---|
| **Probing** | **Definition** | Asking an **"information seeking" question based on information students have verbalized or recorded** about their understanding of the task, mathematical representation of the task, mathematical work, or mathematical statements. |
| | **Examples** | *What did you guys come up with?*<br>*Where did you get these numbers from?*<br>*Show me how you set this up?*<br>*Does this match up with what is labeled on your triangle?*<br>*what are you guys going to do to help solve this problem?*<br>*What do we have to do before we solve for x?*<br>*Why did you cross multiply?* |
| **Pressing** | **Definition** | Asking a question or making a statement that encourages students to **explain or justify their reasoning beyond their initial explanations, to think more deeply about a mathematical idea, or extend their thinking to a new idea related** to their understanding of the task, mathematical representation of the task, mathematical work, mathematical statements, or other students' contributions. |
| | **Examples** | *Can you use the same idea, or do you have to use something different?*<br>*Try setting it up a different way and see if you get the same number or a different number.*<br>*Can we verify that this uses the least amount of zip line wire?*<br>*Can you find some more solutions to see if that is the best solution or not?*<br>*Can you find some math to back up what you are saying?*<br>*If we think about this as an absolute value function, how is that going to help us figure out the location of the island?*<br>*Is there a way we can show algebraically what is happening in the table?*<br>*How could we take this and write a rule?*<br>*Is there a way to prove mathematically what you just said?*<br>*How could you prove or disprove what she is saying?*<br>*How do you know this rectangle you created is the biggest area?* |

| | | |
|---|---|---|
| **Orienting** | **Definition** | Asking a question or making a statement that encourages students to **hear, use, or connect a student's or class idea or questions to their own idea** related to their understanding of the task, mathematical representation of the task, mathematical work, mathematical statements, or other students' contributions. |
| | **Examples** | *How about Carla, does she have the same picture as you?*<br>*So, talk to each other about why you chose Pythagorean Theorem.*<br>*Caleb take your idea and apply it to her picture.*<br>*Okay you have two ideas, she said set up to cross multiply and you said Pythagorean theorem...*<br>*Turn and talk to your groups about how you would solve this problem.*<br>*Each of you compare your numbers with each other.*<br>*Jacob, as she is drawing, can you tell us what she is putting up there and what it represents?*<br>*Do you mind showing that work you just talked about on the side of your paper, so you can see where it can from, so they can see it and you can explain it to the rest of your group?*<br>*Kamin, can you share what you are working on with the rest of your group?*<br>*Jalen make sure he understands where your numbers are coming from.* |
| **Explaining** | **Definition** | **Making a statement to explicitly clarify to students** an aspect related to the task, mathematical representation of the task, mathematical work, mathematical statements, or other students' contributions. |
| | **Examples** | *Minimize, it means the least amount of wire is going to be used.*<br>*That is if you are dividing in half.*<br>*Break this up into 2 pieces x and 600-x.*<br>*This is a right triangle, and this is a right triangle.*<br>*Equal means congruent or the same.*<br>*If you do it on one side, then you have to do it on the other*<br>*Go back and read the problem again.*<br>*Include that in your picture.*<br>*We are trying to minimize the length of the wire and we need these distances.*<br>*The only thing that will vary is the island location.*<br>*The towers are set. That is the height.*<br>*The shape of the wire is not the function.* |
| **Revoicing** | **Definition** | **Restating a prior students' prior contribution by repeating or rephrasing** statements related to the task, mathematical representation of the task, students' mathematical work or thinking, or students' mathematical statements. |
| | **Examples** | *These statements will be in direct response to a students' statement and will thus be a repeat or rephrase of what they said related to the task, mathematical representation of the task, students' mathematical work or thinking, or students' mathematical statements.* |

# APPENDIX E

# LESSON SUMMARY EXAMPLE

| | Launch | Monitor | Discuss | Debriefing Notes |
|---|---|---|---|---|
| **Fall 2015** | 10 minutes<br><br>ARQ: 4<br>TP:0<br>TO:4<br><br>Pressing: 1<br>Orienting: 15<br>Probing: 47<br>Revoicing: 23<br>Explaining: 9<br><br>***Summarized as probing and revoicing student's ideas, questions, and approaches about the task to ensure students could productively get started on the task.***<br><br>**Moves/Goals:** She used moves for goals that achieved the purpose of launching, maintained the demand of the task, and ensured students can get started by using probing and orienting moves.<br><br>**Shift in Practice:**<br>In this launch, she did not address the diagram or mathematical notation thus maintaining the cognitive demand of the task. Similar to previous launches | 33 minutes<br><br>ARQ: 4<br>TP:3<br>TO:3<br><br>Pressing: 21<br>Orienting: 8<br>Probing: 98<br>Revoicing: 23<br>Explaining: 28<br><br>***Summarized as probing and using a orienting moves to support students in developing a common representation and mathematical approach and pressing moves to support students to using their approaches to think more deeply about mathematics in relation to the goal of the task.***<br><br>**Moves/Goals:**<br>The majority of her ***orienting*** moves were to get groups to try different values and compare them to move toward the mathematical goal. She oriented and connected students to one another's mathematical thinking twice to ensure that all group members could engage with the task with a common understanding.<br><br>The majority of her ***pressing*** moves were to perturb students' partial understanding of the mathematics of task by pressing them to reason about the mathematics they were using and to engage more deeply in the mathematics of the task.<br><br>**Shift in Practice:**<br>Different that the first lesson, she allowed students to engage with their conjectures as she probed and used pressed for justification while helping move them toward the mathematical goal. In addition, she spent less time explain procedures, but this may have been an issue related to differences in students' mathematical knowledge. In addition, the nature of her explaining moves were very different. In the first lesson, they were focused on explaining mathematical procedures, in this lesson, they were focused on explaining mathematical elements of the representation and goal of the task so students could apply their approach/procedures toward the mathematical goal. | 27 minutes<br><br>ARQ: 4<br>TP:2<br>TO:3<br><br>Pressing: 2<br>Orienting: 3<br>Probing: 35<br>Revoicing: 9<br>Explaining: 14<br><br>Two students shared and explained their work. The first student tried multiple data points such as 300,300 or 400, 200. The 2nd student used variables x and 600-x to represent the various distances.<br><br>***Summarized as orienting the class to a students work to probe and explain the student about their work while revoicing and probing the class for understanding and verification, pressing the class toward the mathematical goal and guiding a student by probing and explaining their work to build a mathematical function to solve the task.***<br><br>**Moves/Goals:**<br>Mostly factual or IRE questions, however, she does ***orient*** students to the two approaches and link them together, using them to demonstrate building a function.<br><br>She frequently ***pressed*** students to provide evidence or back up their claims.<br><br>**Shift in Practice:**<br>Similar to first lesson. She seems to consistently leverages students work in discussions to build toward her goal. | "I was very pleased with what I was able to pull out of the kids, what they could figure out on their own and the way they handled group work"<br><br>"I ask a lot of why questions and "why" questions support their answers"<br><br>"I think I was much more attentive to the specific things I needed to do, instead of making it happen by fronting loading information"<br><br>"I was more conscious of it because of [the PD]…Revoicing, wait time, questioning styles, facilitating vs. instruction. I'm experiencing some growing pains from it, but that's where I am learning and changing what I am doing in the classroom"<br><br>"I know that has to do with rehearsing and practicing this summer. Just the more you do it, the more comfortable you get with it." |